

# IMO 2010 CMS REPORT

## Report on the 51<sup>st</sup> International Mathematical Olympiad, in Astana, Kazakhstan

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The Canadian IMO team has recently returned from an exciting Olympiad in Kazakhstan, bringing back unique memories, a strong team bond, and an incredible 13th place out of 96 countries, one of the best results ever in Canadian history. This year's team consisted of Robin Cheng, Pinetree Secondary School (Coquitlam, BC); Alex Song, Detroit Country Day School (Detroit, MI); Hunter Spink, Western Canada High School (Calgary, AB); Chen Sun, A.B. Lucas Secondary School (London, ON); Yuqi Zhu, University Hill Secondary School (Vancouver, BC); and Jonathan Zung, University of Toronto Schools (Toronto, ON). Accompanying them were team leader Adrian Tang and deputy leaders Victoria Krakovna and Alexander Remorov. It was a privilege for the three of us to share the IMO experience with these amazing students, supporting them during the challenging training and competition, and enjoying great times together.

### Training camp

The team spent a week training at Wilfrid Laurier University in Waterloo, and a fair amount of training had also been done online before the training camp started. On a typical training day there was a mock olympiad in the morning, a lecture in the afternoon, and a problem-solving session in the evening. Each of the three leaders gave two lectures over the five days of the camp, and one of the days actually contained two lectures (by the Pigeonhole Principle), so the training was fairly intense. Even so, the students continued doing problems during meals, and worked in the classroom long after the problem-solving session officially ended. No worries, there was time for relaxation as well - every evening there was a card game of Mafia, and the camp ended on a recreational note with a movie night.

While we were doing the training, Dr. Christopher Small (University of Waterloo) and Dr. Edward Wang (Wilfrid Laurier University) were around to help with the organizational part of the camp and to give the team valuable advice and encouragement. Two IMO alumni came by to have fun with the team and enjoy the problems - Elyot Grant and David Rhee, who now study mathematics at the University of Waterloo. One day Dr. Small brought a very pleasant surprise - a replacement for the team mascot Canmoo, who was lost on the way back from the 2009 IMO. The new Canmoo is a cute miniature moose, who shows his Canadian pride by wearing a red sweater that says "Canada", and, more subtly, with a small Canadian flag tattooed on his foot. By the end of the camp, the team had all the ingredients for an excellent performance at the IMO - strong students, a great week of training, and a new mascot.

### Vienna

On their way to Kazakhstan, the team had a 12-hour layover in Vienna. Though many of the students did not get a good sleep on the flight, they were eager to explore the city.

Getting to and around downtown Vienna was easy, thanks to the generally good navigability of the place and some instructions from Gertrud Jeewanjee of the CMS, a native Austrian. After spending some time in a souvenir shop (the students were particularly excited by the rich variety of music boxes), the team went to the central square, the site of the majestic St. Stephen's Cathedral. They arrived there during a mass and had a chance to enjoy the excellent acoustics of the cathedral.

The group continued walking around the beautiful downtown, making several stops for refreshments due to the hot weather and tiredness (carrying around their hand luggage did not help). When they walked into a pleasant air-conditioned museum, the students just sat down on the floor near the entrance, putting their bags all over the place. They were so relaxed that it took ten minutes to get them out of there, and head for a more appropriate location. That in fact existed nearby, and in an unexpected place too - the garden next to the Hofburg Imperial Palace, which looked funny with crowds of people sunbathing everywhere. In front of the Palace there was a statue of Mozart, which allowed Chen to show his statue-mimicking skills. The last stop was the Parliament building - the team did not go inside, but had a good time hanging out on the front steps in the company of great philosophers. When it was time to head back to the airport, everyone was quite tired but pleased with the excursion.



### Problem selection in Almaty, Kazakhstan

Adrian left the training camp a few days early to attend jury meetings on problem selection, and arrived in Almaty at midnight local time. The team leaders were immediately greeted warmly by local Kazakh organizers. Most did not speak much English beyond "Welcome to Kazakhstan", but their warm smiles assured the leaders that they will make their stay enjoyable. Exhausted, Adrian checked into his comfortable room, showered and promptly fell asleep.

As an interlude, here are some simple Kazakh phrases:

Salem - Hello (informal)

Saubol - Good-bye (informal)

Rahmet - Thank you

Menin atym - My name is

Salem, our dear CMS Notes readers. You now know just as much Kazakh as the Canadian team.



Upon waking up, Adrian discovered that the hotel was in fact a relaxation and wellness centre, with mineral water baths. The hotel was surrounded by a beautiful park in which people could ride bikes, paddle boats and exercise. His balcony overlooked a driving range surrounded by beautiful greenery, with a scenic mountain backdrop. It was a great sight to wake up to in the morning.

The team leaders received the shortlist problems, from which six problems were to be chosen for the competition. Their job was to judge the beauty and difficulty of as many problems as they could over the following two days. Adrian gathered with fellow leaders to solve and discuss the problems, and promptly solved the easy problems. He noted two nice and easy problems, a functional equation and a geometry problem. The Canadians had a thorough session on functional equations and were overall quite strong in geometry. Hence, Adrian's choices for the favourites were clear.

The leaders had an afternoon excursion to Medeo, the site of the future Asian Winter Games in 2011. Although still under construction, the skating rink facilities appeared to be at a world class level and the mountains were glorious. The group hiked up the steps to the top of the mountain and soaked in the picturesque views from the top.

A jury meeting was scheduled for that evening at 8pm. However, the German leader spoke up and reminded the jury that the Germany vs Argentina World Cup game was at 8pm that evening and proposed to shift the meeting. This was met with thunderous applause from everyone. Even the jury chair jokingly apologized for his "scheduling over this

important event". The priorities became World Cup > IMO Shortlist. The game was projected onto the big screen in the jury meeting room and roughly 100 leaders and observers gathered to watch the game. Watching with an international crowd was a very unique experience and definitely made the game far more exciting than usual.

The shortlist problems were of very high quality. Consequently, the jury had many choices for the medium and difficult problems. Adrian was particularly enamoured with the following combinatorics problem:

Problem 5. In each of 6 boxes  $B_1, B_2, B_3, B_4, B_5, B_6$  there is initially one coin. There are two types of operations allowed:

Type 1: Choose a non-empty Box  $B_j$  with  $1 \leq j \leq 5$ . Remove one coin from  $B_j$  and add two coins to  $B_{j+1}$ .

Type 2: Choose a non-empty Box  $B_k$  with  $1 \leq k \leq 4$ . Remove one coin from  $B_k$  and exchange the contents of  $B_{k+1}$  and  $B_{k+2}$ .

Determine whether there is a finite sequence of such operations that results in boxes  $B_1, B_2, B_3, B_4, B_5$  being empty and box  $B_6$  containing exactly  $2010^{2010}$  coins.

We ask you, the reader, to play around with this fun problem. Go ahead. Come back in an hour after you've played with it.

Okay, you are back. Do not read this paragraph if you do not want spoilers. Would you believe that the answer is yes? Would you believe that you can obtain far more coins than what is asked in the problem? Let  $P_1 = 2$  and  $P_{n+1} = 2^{\{P_n\}}$  for all positive integers  $n$ , i.e.  $P_n$  is the value of a tower of exponents containing  $n$  twos. Would you believe that you can obtain  $2 \times P_{14}$  coins? How about  $2 \times P_{\{P_{14}\}}$  coins? The answer to all of the above is yes. This result is absolutely mind-blowing. We strongly recommend you to share this problem with your students in your math clubs, math circles or math classes. We promise you it will be worthwhile.

This problem narrowly won over another combinatorics problem. Adrian was happy that all six chosen problems were amongst his favourites. The final stage was formulating the wording of the problems in English and the subsequent translations into over 50 languages, which traditionally is a big hassle. Thankfully this year, the English version required only minor tweaks in five of the problems and the creation of a cleaner scenario in the aforementioned coins problem to complete the English translation. It is amazing that certain English words cannot be translated easily into some languages. For example, a certain language apparently did not have a word for "stack". After the translations of all languages were completed and approved, there were other orders of business for the leaders, including the election of a new chairman and two new members. Nazar Agakhanov, the Russian leader, was elected chairman. Past chairman József Pelikán's efforts will not be forgotten, as the success of the

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IMO today is primarily due to his vision and hard work. We all owe József a lot of gratitude.

Now it was time to head to Astana, the capital of Kazakhstan, for the opening ceremony and coordination. To Adrian's recollection, this was the first time the leaders were flown from the leaders site to the coordination site. The leaders said goodbye to Almaty, and boarded their 3am flight...

## Opening ceremony and contest days

When the team landed in the Astana airport, some were concerned that they would be taken to the contest location (a children's summer camp) right away, meaning a 4-hour bus ride right after the tiring flight. Fortunately, this was not the case - the newly arrived teams were taken to a luxurious hotel in Astana, and a whole day of relaxation was ahead. The hotel was packed pretty much to maximum capacity - it was amusing to see fancy rooms lined with little wooden beds that were obviously put there just to accommodate the sheer numbers of students.

Astana (literally meaning "capital" in Kazakh) was a very modern city, built only in the past 12 years since the capital was moved there from Almaty. The opening ceremony took place at the Palace of Independence, and coincided with the anniversary of Astana being the capital and the President's birthday. The IMO participants were given a grandiose welcome by lovely Kazakh musicians in national costumes. The team leaders were already at the Palace - Adrian was admiring the murals of the Astana 2030 project, a plan by President Nursultan Nazarbayev to transform Astana into a futuristic looking metropolis by 2030. The team leaders were, of course, separated from the students and deputy leaders during the ceremony, but Adrian and his team still exchanged waves of hellos from a distance.

The opening ceremony began with children coming on stage wearing t-shirts with math symbols on them. It was an adorable sight that melted the audience to go "awwwwww...". The organizers went all out to provide lively musical and dance performances for everyone to enjoy. During the introduction of countries, the Canadian team walked across the stage proudly, waving the maple leaf flag and holding on to Canmoo, the moose that gives them strength, loved by all and stolen by many.



After the opening ceremony, the deputy leaders and students departed to the contest site at the Baldauren camp, and the leaders returned to their hotel in Astana, in preparation for the meeting to approve the marking schemes for the problems. Here is a tip for everyone: never create and discuss marking schemes when you are sleep deprived. The team leaders' day began at 3am and it was decided to continue the jury meeting on the following day after everyone had slept. However, this did not deter many of them from watching the Netherlands vs Uruguay World Cup semi-final game that night at 12:30am.

The next day began with the Question and Answer period. Students' questions are faxed to the leaders' room, and each student's leader proposes an answer to return to the student. Leaders braced themselves that no embarrassing questions come from their own students. Gems from past years (not from Canadian students) included "What is a grasshopper?" and "I cannot draw the diagram (described in the problem). Can you do it for me?" This year's questions were fairly straightforward and uneventful, to the relief of everyone.



The leaders went on an excursion to downtown, the site of the Baiterek Tower, the symbol of Astana. Downtown Astana encapsulated a futuristic look, with gold-coloured buildings and modern architecture that rose to the sky, some in the shape of a parabola. At the base of Baiterek was a square with an impressive display of bear statues, one for each country in the world. The bears symbolized love and unity amongst nations. The bear representing Canada was disappointing - it was blue-and-white and pixelated. Adrian could not fathom why this represented Canada. Other bears were more colourful and symbolic of their representing countries. At the top of the tower was a small podium with the handprint of the President of Kazakhstan. Visitors were encouraged to place their hand in the handprint and make a wish. Adrian did, and wished that the Canadian team would have a wonderful time in Kazakhstan.

Meanwhile, the deputies and students were enjoying the beautiful Kazakh countryside. The Baldauren camp, well-known in Kazakhstan as the Island of Children's Dreams, was a very nice facility that gave the students an opportunity to play sports, go boating on the lake, and enjoy nature. Unfortunately, the deputies were staying at a different place, a resort a half-hour drive away, which was also a very picturesque place, but without easy access to the students. Thankfully, they were taken to Baldauren after each half of the contest to spend time with the students and discuss how they did. The deputies enjoyed a great excursion to the Burabai National Park and an exciting sing-and-dance party called the "Night of Surprises", and then it was time to return to Astana for coordination. Meanwhile, with the contest worries behind them, the students stayed at Baldauren for three more fun-filled days.



*Canadian team in Burabai National Park with their guide Zauresh*

### Coordination

After Day 1, many members of the Canadian team showed their usual modesty proclaiming "they failed" on the first day. Leaders often have to remind students that their performance is always relative to other countries and to trust the leaders in their job of obtaining as many marks for each student as possible. Adrian received the Day 1 solutions and was delighted to know that Robin seemed to have a perfect Day 1. On Problem 1, Canada had three perfect solutions, one solution with a minor flaw, and two incomplete solutions which were still worth five (out of seven) points. On Problem 2, Canada had two perfect solutions, but a plethora of part

marks amongst the other four solutions. The problem was a tricky geometry problem, and hence, the Canadian leaders were content with this result. Canada had one perfect solution to Problem 3, a difficult number theory problem disguised as a functional equation, and also three solutions with partial progress, which eventually yielded Canada part marks. Hooray for partial marks!

Adrian received the Day 2 solutions the day after, and was happily reunited with Victoria and Alex. Woohoo! We scoured the Day 2 solutions and were further delighted that Hunter seemed to have a perfect Day 2. Canada had five perfect solutions to Problem 4, two to Problem 5 and one to Problem 6. Unfortunately several students on the team thought the answer to problem 5 was no and spent most of their time on Day 2 trying to prove a false claim. However, this misfortune fell not only upon Canada, but on every other country, so all was not lost for Canada. Two of our students received full marks and two obtained partial marks on Problem 5. More hoorays for partial marks!

Coordination went very smoothly with the exception of one problem, namely Problem 1, where the coordinators insisted on going through every line for five of our students' solutions. This is a warning to future team members: the coordinators wanted to deduct marks for minor things such as typos and bad notation on Problem 1. Hence, please write solutions to easy problems with extreme care. Canada narrowly escaped punishment this year, but some other countries were said not to be as lucky. It was very beneficial at this problem coordination to speak Russian - while the coordinators did have good English, they were more comfortable with Russian, and spoke it among themselves. The assertiveness and Russian skills of Alex and Victoria were pivotal in preventing the coordinators from deducting marks in places not deserving such mistreatment. For the remaining five problems, we agreed on the scores with little to no conflict. There was one mishap where a student used the word "obvious" in his solution, which caused some minor headaches since his claim was anything but obvious. Oh, the dreaded "o" word. Thankfully, this issue was resolved when Alex spent 20 minutes explaining (in Russian) why the student's claim was indeed obvious.

After coordination, Canada's score was 129 out of 252 and we looked to be ranked among the top 20 countries. This itself would be a great success for Canada already. Robin and Hunter seemingly had gold medals secured with scores of 35 and 32, respectively, and Yuqi had his bronze medal secured with a score of 19. Jonathan received an honourable mention, with a performance worthy of recognition. Chen's score of 21 and Alex's score of 15 were dancing on the edge of the silver and bronze cutoffs, respectively. In each of the past three years, Canada had at least one student who was one mark short of a cutoff. Oh please let the universe balance and give Chen and Alex the desired medals! The jury meeting began and the medal gods responded - the gold, silver and bronze cutoffs were

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determined to be 27, 21, 15, respectively. Hi-five! Silver for Chen and bronze for Alex! It is worth noting that Alex is only 13 years old, which makes his bronze medal achievement even more astounding. This resulted in a total of 2 golds, 1 silver, 2 bronzes and an honourable mention for Canada. More exciting news arrived, as we discovered that Canada was ranked 13th! Thinking 20th and receiving 13th set off all sorts of fireworks in our heads. Percentile-wise, this is Canada's best performance ever! We could not handle any more news, until we realized that individually, Robin and Hunter ranked 7th and 11th, respectively, out of all 517 contestants. Wow Wow Wee Wow! Smiles and hugs were shared as we celebrated, laughed and giggled throughout the night. We emailed the results to the students and to interested parties back in Canada and could only imagine the elation and glee from everyone.

## The last days and farewell

After the coordination, it was time for the leaders to sit back and have some fun. We enjoyed the bowling and go-carting facilities at the hotel, and several excursions - to the Cultural Centre of Astana, the President's Museum, and the "Miniature Kazakhstan" complex. Finally, the big day arrived when we were reunited with the students, who returned to Astana from Baldauren. Together, we went to see an amazing equestrian show of Kazakh national sports. The performers did all sorts of incredible acrobatics on horses - riding two horses at once, swinging off the horse at full gallop to pick up a handkerchief from the ground, wrestling each other off their horses, and so forth. There was also an adorable game where a guy chases a girl on his horse, in the attempt to hug and kiss her while in full gallop. If he fails, then on the way back she chases and whips him!

Afterwards, the students spent a day enjoying excursions around Astana, and then it was time to return to the Palace of Independence for the closing ceremony. The students were given the red carpet treatment, literally, as they marched on a red carpet into the Palace to a majestic fanfare performed by Kazakh musicians. The leaders followed soon after and the closing ceremony began. We cheered on as the Canadians proudly went up on stage and received their medals. The gold medalists were further rewarded with laptops, courtesy of the main sponsor of this IMO, ExxonMobil.

After the ceremony, we celebrated our glory and achievement this year. We bussed back to the leaders' hotel for the farewell dinner, and were joined at our table by the Canadian guide, Zauresh. She did an amazing job taking care of our team. The Canadians loved her, and she loved the Canadians. We proudly signed our names onto a Canadian flag and presented the flag to her as a gift. Zauresh's eyes lit up in happiness. The dinner consisted of an assortment of Kazakh cuisine, which we feasted on to our hearts' delight.

We were only hours away from departing from Kazakhstan. An event is only as wonderful as the people who were there to share the experience with. We said farewell to our friends and colleagues, who filled our experience in Kazakhstan with happiness, laughter and life-long friendships. We will certainly see them again at a future IMO or through other endeavours. The Kazakhs were wonderful hosts, who made the strongest effort to please out of all the hosts we have ever encountered. Rahmet Kazakhstan, for a lovely IMO 2010.

