Report on the 2005 International Mathematical Olympiad in Merida, Mexico
by Elyot Grant

Attending the 46th annual International Mathematical Olympiad in Merida Mexico has been one of the most unique and memorable experiences of my life. Not only was I able to take part in a prestigious international competition, but I was also given the opportunity to connect with other mathematics students from around the world while experiencing the Mexican culture first hand. In this report, I hope to share some of these remarkable experiences from the perspective of a contest participant.

The International Mathematical Olympiad (IMO) is one of a family of annual academic olympiads for high school students (the others are in biology, physics, chemistry, computer science, and astronomy). Each country may send up to six participants, who must be of age 19 or less and attending high school. This year there were about 90 countries participating. The contest itself consists of two papers written on two consecutive days, each with three difficult mathematical problems and a time limit of 4.5 hours. The questions demand creativity and ingenuity in their solutions and require detailed justification and complete explanation for full credit. The competition takes place during mid-July annually and each year, the contest takes place in a different city somewhere around the world.

Canada offers a wide variety of contests available for students at various grade levels. The top-scoring students can be invited to a series of training camps, which help prepare students for the tougher math competitions. A handful of the top students in Canada are given an invitation to write the Canadian Mathematical Olympiad--Canada's toughest contest--and some of the top contenders write the USA and Asian-Pacific Olympiads as well. The scores and rankings from these contests are tabulated and evaluated by the team selection committee, and Canada's six students for the IMO are chosen.

In June, we flew out to Banff, Alberta for a two week IMO training camp, which consisted of daily lectures, problem solving sessions, and mock Olympiad papers. The training sessions were instrumental in helping the team learn various advanced problem solving techniques (which we actually used when writing the IMO paper). They also provided a little bit of a confidence boost. For once, I could actually see myself solving problems of IMO difficulty. The training camp was perfect preparation for the IMO.

We left Banff after training feeling optimistic and ready for the competition. A few flights later, we were in Merida, all set for the contest. The first evening and second day had few activities planned, but were instead set aside to give the teams a chance to lose their jetlag and mentally prepare for the contest papers. We met up with some other teams, played games, and really just chilled in our hotels for a day. We were bussed to an old Mexican theatre during the second night, and opening ceremonies were held. After a few welcoming speeches given in Spanish (a language that only our team leader, Dr. Felix Recio, could understand), each team was given a chance to walk on the stage and receive a few seconds of applause. Some teams goofed around on stage, others threw out pins and gifts, and many simply walked across the stage. It was a nice way to start the contest.

Although the IMO is about nine days long, the contest is always written early during the week to allow enough time for the papers to be marked and coordinated. Days three and four were contest days. Each contest started at 9:00 in the morning and finished at 1:30 in the afternoon, so we had to wake up quite early to get ready. The paper itself was written in three large rooms of a convention centre, with two members of each team being seated in each room. Each contestant was given a large desk to work on, an enormous stack of answer paper to write on, a boxed lunch to munch on during the contest, and large signs that we could hold up to request a bathroom break or a water bottle. The four and a half hours really seemed to just flash by, as solving Olympiad-level problems can get quite intense.

The problems for the IMO are chosen in a very complicated selection process. Potential problems for the IMO are submitted by different countries and assembled into a long list, and from them, a shortlist of about thirty problems is selected by the IMO committee. After this, the problems are voted on until the final six are chosen for the olympiad paper. The problems are designed to be of varying difficulty level and from many different branches of mathematics (including algebra, geometry, and such). I'll give a brief description of each of the six problems, so the uninterested can skip the next few paragraphs.

Question one was an interesting problem in that it was selected to be the easiest problem on the paper, but turned out to be very difficult to solve for many contestants. It was a standard geometry problem involving a few triangles, and it had a very simple, elegant solution using vectors that many students failed to see. There were many other complicated solutions using geometric transformations, spiral similarities, and trigonometry, but these were quite

difficult to obtain and often required several pages of arguments. Although the vector solution was simple and almost obvious, it was overlooked by a number of participants, probably because vectors are not commonly used in the solution of olympiad-level geometry problems (most of our training materials had us solving problems using much more complicated techniques). Fortunately, most of the Canadian team solved this problem somehow, although a few of us lost points because of messy or incomplete solutions.

The second problem was definitely the easiest of the first day, and was a simple sequence problem that could be solved using deductive arguments. There was really only one solution to this problem, and it was a simple, step-by-step analysis of given facts. Most of our team did quite well on this one.

The third problem was the toughest challenge of the entire olympiad, as few contestants were able to solve it completely. It demanded the proof of a cyclic, three-variable inequality. I was actually surprised that this question was even selected for the IMO, as such problems usually can be solved in an hour or so without much ingenuity using a technique known as Muirhead-Bashing (named after the algebraic formula--the Muirhead Inequality--which is used). Muirhead-Bashing is a form of what us Canadians like to call "grunging", which means solving a problem in an ugly, tedious, long, and inelegant way, but still getting the job done (Americans refer to this technique as "dumb-assing"). Another form of grunging is the use of messy coordinates or complex numbers to solve geometry problems. Muirhead-Bashing has recently gained a lot of popularity (especially on the Internet) as a technique used to solve cyclic 3-variable inequalities. The Muirhead solution worked perfectly on this problem, as long as the student writing up the proof made no errors in the multiple pages of algebra that needed to be worked out. I actually know of students who have invented their own notation to help make Muirhead-Bashing less messy. A few members of our team who were experienced in this technique gave it a try, and two solved the question (although one lost some marks because of an error). The rest of us didn't do so well. There were, in fact, some very elegant solutions to this problem using other well-known inequalities, but few students managed to find them.

Problem four was an interesting number theory problem that had only one solution that was very easy to understand and explain, but relied on one small but crucial observation that was not so easy to see. This was one of those questions where you either see it, or you don't. Most of the scores on this problem were either perfect, or close to zero.

Question five was a geometry problem that was much tougher than the geometry problem of the previous day. Given a quadrilateral with a few known properties and moveable line on it, we had to prove that a certain family of circles all passed through a common point. This problem didn't appear to be very difficult at first glance, but it turns out that the biggest challenge of this problem is determining exactly where this common point lies. A variety of solutions to this problem were given, and most of them weren't too complicated, but this problem was indeed quite tough. A few members of our team did quite well on this one, but a lot of us got zero.

The sixth problem was a combinatorics problem that had a tough, complicated, and long solution. Given an unspecified number of students writing a contest of six questions and a statement about how many times each pair of problems was solved, we had to prove that more than one student solved five problems. The strangest part about this problem was that the statements given in the problem were seemingly unconnected to the result that we had to prove. It was easy to make discoveries about the problem from the given information, but one never knows whether such discoveries will actually lead to a solution. Many contestants wrote over ten pages of solution for this problem, sometimes receiving very few, if any, marks for their work. The official solution involved a bit of modular arithmetic, and the proof itself could be either worked out combinatorially or solved algebraically using linear systems.

For those interested, a much more detailed analysis of each problem and its solution can be found online at many websites such as **www.mathlinks.ro** and **www.artofproblemsolving.com**.

After each day of contest writing, we would each sit down with our deputy leader Dorette Pronk and observer Adrian Tang and discuss our solutions to the day's problems, so that they could prepare to defend our solutions when confronted by the coordinators that evaluate us. The next few days of the olympiad were quite stressful for many of the contestants, as we eagerly awaited to find out how many points our solutions would receive. Even after we knew the scores, we still couldn't relax, because we didn't know what the cutoff scores would be for gold, silver, and bronze medals (which are given in a ratio of approximately 1:2:3, with about half of all participants receiving medals).

After the contest days, the Olympiad is really over for the contestants, so we had a lot of time to relax and hang out with olympians from other countries. We went on many tourism excursions to the Mexican pyramids and beaches, which were quite a lot of fun. We also played a lot of games like soccer, table tennis, and foosball. We even played card games like poker, mafia, and mao to pass the time. Of course, many contestants were eagerly discussing the problems and their solutions, and many new friendships were made.

The most unusual aspect of this year's IMO was the hurricane that struck Merida late in the second week. Hurricane Emily forced us to cut our travel time short, pack up all of our belongings, and stay in the basement of the hotel for a day until it passed. The hotel staff taped up all of the windows and tied all of the cabinets shut. We actually had a lot of fun at what became known as the "hurricane slumber party". Most people stayed up all night playing cards and games and just chilling with the other IMO people. Fortunately, the worst part of the hurricane went north of Merida, and our hotel rooms stayed in good shape.

Fortunately, we were still able to have our closing ceremonies, which involved a few farewell speeches and the presentation of the medals. Canada received one gold, two silver, and two bronze medals, and we ranked 19th overall. The final evening was of course a huge party, and quite sad in a way, because many of us had to say goodbye to our new friends from faraway countries.

Overall, the IMO was amazing. I would definitely do this again, even if I wasn't competing. Of course, a huge thanks goes out to the Canadian Math Society and all of their sponsors, namely: the Samuel Beatty Fund; the provincial governments; SunLife Finacial; the research institutes and the schools and teachers who make this possible, as well as the many profs and students who work to train Canadian students for the competition. It's a remarkable, unforgettable experience that has left me with many fond memories and lasting friendships. It truly has been an honour to have been selected to attend such a fantastic event.