

Focus On ... Index

Algebra

1 Integer Part and Periodicity

38:3 (Mar 2012) 99-100

A periodic function that vanishes over a period is the zero function. This obvious property can lead to elegant proofs for some identities involving the integer part function.

3 From Linear Recurrences to a Polynomial Identity

38:7 (Sep 2012) 276-277

To determine the set of all sequences U_n satisfying the linear recurrence $U_{n+2} = xU_{n+1} - yU_n$, a direct approach (instead of the classical method) leads to a general polynomial identity.

7 Decomposition into Partial Fractions

39:5 (May 2013) 218-221

Some examples involving minimal calculations to display the usefulness of this algebraic tool.

15 A Formula of Euler

41:1 (Jan 2015) 16-20

Euler observed that the sum $S(n, m) = \sum_{k=0}^n (-1)^{n-k} \binom{n}{k} k^m$ equals 0 if $m = 0, 1, \dots, n-1$ and equals $n!$ when $m = n$.

17 Congruences (I)

41:5 (May 2015) 202-205

In many problems involving integers, an ingenious appeal to congruences can be most helpful. Choosing the appropriate modulus is often the key to a short solution.

18 Congruences (II)

41:7 (Sep 2015) 295-298

Here, the focus is on congruences modulo a prime number, with emphasis on the consequences of three simple, but useful, theorems.

23 Vieta's Formulas

42:7 (Sep 2016) 303-307

The formulas that relate the coefficients of a polynomial to sums and products of its roots can help solve algebra problems and establish inequalities.

25 The Long Division of Polynomials

43:1 (Jan 2017) 16-20

The division algorithm for polynomials can provide the key to solving a variety of algebra problems.

26 Degree and Roots of a Polynomial

43:5 (May 2017) 205-209

The focus here is on the links between the degree of a polynomial and the number of its roots.

30 Cauchy's Functional Equation

44:3 (Mar 2018) 106-109

Solutions to problems involving functional equations often come down to an application of known results about Cauchy's equation. A selection of problems illustrate the various properties under consideration.

Inequalities

5 Inequalities via Lagrange Multipliers

39:1 (Jan 2013) 24-26

A few examples of problems requiring the proof of a constrained inequality where, with some care, the method of Lagrange Multipliers leads to a simple solution.

20 Inequalities via Complex Numbers

42:1 (Jan 2016) 20-23

Geometric and algebraic inequalities follow from familiar properties of the modulus of a complex number.

23 Vieta's Formulas

42:7 (Sep 2016) 303-307

(See under Algebra.)

40 Inequalities via Auxiliary Functions (I)

46:3 (Mar 2020) 117-122

We illustrate through examples how to verify a given inequality by choosing an appropriate auxiliary function, then using calculus.

Geometry

2 The Geometry Behind the Scene

38:5 (May 2012) 183-185

An algebra problem can sometimes be simplified if a connection to a corresponding geometry problem can be found.

4 The Barycentric Equation of a Line

38:9 (Nov 2012) 367-368

A geometric look at the coefficients of the barycentric equation of a line in the Euclidean plane, and some applications.

6 Glide Reflections in the Plane

39:3 (Mar 2013) 133-135

A few situations where glide reflections provide insight.

8 Generalized Inversion in the Plane

39:7 (Sep 2013) 307-310

We extend the notion of inversion to include the commutative product of a classical inversion in a circle and the half turn about its center. This provides a unique inversion with center O exchanging points A and B no matter the relative position of the three collinear points.

12 Intersecting Circles and Spiral Similarities

40:5 (May 2014) 203-206

Let the circles C_1, C_2 intersect at points U, V . Among the spiral similarities transforming C_1 into C_2 , the one with center U provides a simple way to obtain the image of any point P of C_1 — it is the second point of intersection of the circle C_2 with the line through P and V .

13 The Dot Product

40:7 (Sep 2014) 289-292

The purpose of this essay is to show the dot product at work through alternative solutions to several past geometry problems.

16 Leibniz's and Stewart's Relations

41:3 (Mar 2015) 110-113

Leibniz's relation involves the center of mass of n weighted points in d -dimensional Euclidean space. It deserves to be better known; among other things, Stewart's theorem is an easy consequence.

21 The Product of Two Reflections in the Plane

42:3 (Mar 2016) 109-113

Knowing that the product of two reflections is either a translation or rotation can help prove geometric theorems and solve geometric problems.

22 Constructions on the Sides

42:5 (May 2016) 211-215

We investigate configurations involving triangles or quadrilaterals constructed on the sides of triangles or quadrilaterals, and favor proofs using transformations or complex numbers.

27 Some Relations in the Triangle (I)

43:7 (Sep 2017) 293-297

The goal here is to present a selection of less familiar relations among the parts of a triangle that are attractive and useful.

28 Some Relations in the Triangle (II)

43:9 (Nov 2017) 389-393

A continuation of Number 27, focusing here on formulas involving lengths related to the classical cevians.

32 Harmonic Ranges and Pencils

44:7 (Sep 2018) 291-296

Elementary properties of a harmonic conjugate can lead to simple and elegant solutions to some geometry problems.

36 Geometry with Complex Numbers (I)

45:5 (May 2019) 258-264

We focus here on the use of complex numbers to prove results involving a triangle and its circumcircle.

37 Geometry with Complex Numbers (II)

45:7 (Sep 2019) 407-412

We continue our discussion from Number 36, here using complex numbers to deal with regular polygons, similarities, and areas.

39 Introducing S_A, S_B, S_C in Barycentric Coordinates

46:1 (Jan 2020) 26-31

Barycentric coordinates relative to a triangle are appropriate not just for dealing with affine properties such as collinearity, concurrency, and areas, but they can often be used also for Euclidean properties such as lengths and perpendicularity.

Calculus

5 Inequalities via Lagrange Multipliers

39:1 (Jan 2013) 24-26

(See under Inequalities.)

10 Some Sequences of Integrals

40:1 (Jan 2014) 21-24

A study of the sequences $I_n = \int_0^1 (ax^2 + bx + c)^n dx$ with the goal of finding a sequence (ω_n) such that $\lim_{n \rightarrow \infty} \frac{I_n}{\omega_n} = 1$.

11 The Partial Sums of Some Divergent Series

40:3 (Mar 2014) 112-115

For a sequence (a_n) of positive real numbers such that $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \omega > 1$, and the corresponding sequence of its partial sums $A_n = \sum_{k=1}^n A_k$, we establish a few results about the behavior of the sequences (A_n) and $\left(\frac{A_{n+1}}{A_n}\right)$ in comparison with the sequences (a_n) and $\left(\frac{a_{n+1}}{a_n}\right)$ and offer some applications.

15 A Formula of Euler

41:1 (Jan 2015) 16-20

(See under Algebra.)

30 Cauchy's Functional Equation

44:3 (Mar 2018) 106-109

(See under Algebra.)

31 Mean Value and Rolle's Theorems

44:5 (May 2018) 202-206

The Mean Value Theorem establishes a link between a function and its derivative. Here we will see it at work in various problems, sometimes rather unexpectedly.

35 The Asymptotic Behavior of Integrals

45:3 (Mar 2019) 137-143

We keep the same goal as in Number 10 [2014: 21-24] (namely, to determine the asymptotic behavior of a given sequence of integrals), but here, restricting ourselves to elementary problems and methods, present simple ways to obtain such an asymptotic behavior.

Combinatorics

15 A Formula of Euler

41:1 (Jan 2015) 16-20

(See under Algebra.)

Trigonometry

34 Some Trigonometric Relations

45:1 (Jan 2019) 26-32

We consider a selection of problems involving the values of the circular functions at $\frac{m\pi}{n}$ for various natural numbers m and n . Solutions depend on complex numbers and polynomials in addition to the classical trigonometric identities.

Solutions to Exercises

9	39:9 (Nov 2013)	404-408	From Numbers 2 to 5.
14	40:9 (Nov 2014)	380-385	From Numbers 6 to 11.
19	41:9 (Nov 2015)	386-391	From Numbers 12 to 16.
24	42:9 (Nov 2016)	393-397	From Numbers 17 to 21.
29	44:1 (Jan 2018)	19-24	From Numbers 22 to 26.
33	44:9 (Nov 2018)	377-381	From Numbers 27 to 31.
38	45:9 (Nov 2019)	511-517	From Numbers 32 to 36.

