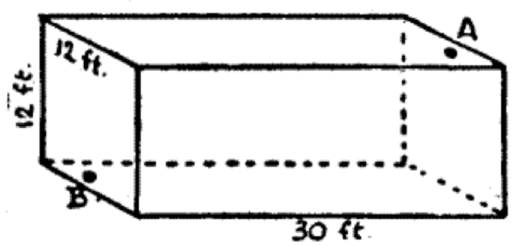
TEACHING PROBLEMS

No.9

Erick Lee

The Spider and the Fly

Inside a rectangular room, measuring 30 feet in length and 12 feet in width and height, a spider is at a point on the middle of one of the end walls, 1 foot from the ceiling, as at A; and a fly is on the opposite wall, 1 foot from the floor in the centre, as shown at B. What is the shortest distance that the spider must crawl in order to reach the fly, which remains stationary? Of course the spider never drops or uses its web, but crawls fairly.

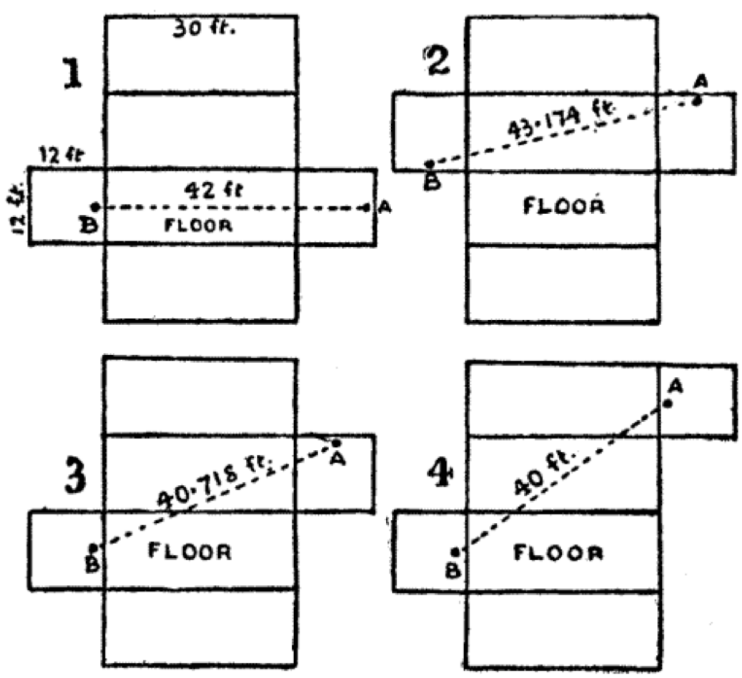


This problem was created by Henry Ernest Dudeney. It is problem 75 from his book *The Canterbury Puzzles* published in 1908. Dudeney was an English mathematician and prolific creator of logic puzzles and recreational mathematics problems. From 1910 until his death in 1930, Dudeney wrote a monthly column in *The Strand Magazine* entitled "Perplexities" which featured mathematical brain-teasers. As *The Canterbury Puzzles* was published over 100 years ago, it is freely available on The Project Gutenberg website at `http://www.gutenberg.org/files/27635/27635-h/27635-h.htm`.

When introducing this problem to students, I draw a spider and a fly each on their own index card and tape them to the appropriate spots on the wall in the classroom which is the shape of a rectangular prism, although not exactly the same dimensions as the given problem. I then describe the problem of the spider and the fly using the classroom to physically model the problem. Students often struggle to visualize problems in three dimensions despite living in a three-dimensional world. Many students will quickly determine that the spider should take the "straight path" directly up to the ceiling (1 ft), directly across the ceiling (30 ft), and down the opposite wall (11 ft). This will give a total distance of 42 ft.

After the majority of the class has come to this conclusion, we have a discussion. I ask them, "How do you know that this is the shortest path?" Students often respond that a straight line is the shortest distance between two points. To challenge their thinking, we discuss how a straight line might look different in three dimensions than in two dimensions. The great circle routes that airplanes fly often seem counterintuitive when students visualize the Mercator projection maps commonly found in classrooms. Where we live in Nova Scotia, we can look into the sky at almost any time of the day and see airplanes high in the sky flying from the Northeastern United States to European destinations. This only makes sense when looking at the great circle route on a spherical globe.

I challenge the class to brainstorm a variety of different routes that the spider might take and to calculate the distances for each of these new routes. To help in their brainstorming, I suggest that they examine possible routes on a net drawing of the room instead of a three dimensional drawing. Some students might find it helpful to model the room with a manipulative which allows them to link polygons together (such as *Polydrons*) on which they could label the walls, floor and ceiling of the room as well as the position of the spider and the fly. This would allow students to see how the path might change depending on how they create the net of the room. You might challenge the students to find a net that results in the spider crossing three sides of the room, four sides of the room or even five sides of the room and to see how these different nets result in different distance paths. Eventually, students will find the solution of the shortest path. Below are four different nets that Dudeney showed in *The Canterbury Puzzles*.

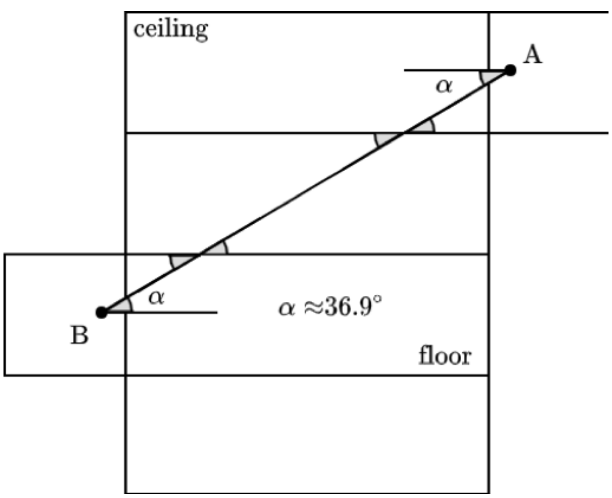


The distances for each of the solutions above can be found by applying the Pythagorean Theorem. Students at times have difficulty imaging the shortest path given in solution number 4. Returning to the pictures of the spider and the fly taped to the walls, a long piece of yarn is used to show the path of the spider along the sides of the classroom.

In his book *The Pythagorean Theorem: A 4,000-Year History*, Eli Maor describes how we could accurately trace the spider's path using trigonometry. In case 4 above, the spider's horizontal distance is 1 + 30 + 1 = 32 feet and the vertical distance travelled is 6 + 12 + 6 = 24 feet.

$$\tan\alpha = \frac{24}{32}$$
$$\arctan\frac{24}{32} = \alpha$$
$$\alpha \approx 36.9^\circ$$

The diagram below shows how the spider travels using this angle across the sides the room.



To imagine why this is the shortest path think about the shape of the room as a cylinder with hemispherical ends instead of a rectangular prism (like a hot dog instead of a block of wood). Imagine the piece of yarn wrapping around this shape from the spider to the fly's position. Now imagine if this "hot dog" shape slowly changed shape, or "deflated", until it was the rectangular prism. The curving path from the cylinder would now be the angled path of the spider around the classroom.

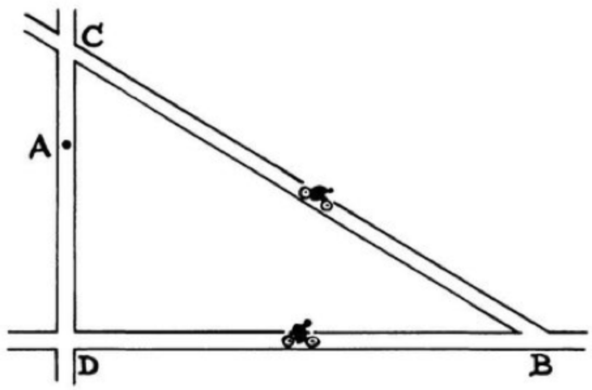
There are several extensions to this problem that could be explored:

- How do the dimensions of the room affect the shortest path that the spider takes? Would the path from the problem we solved be the shortest path for a room with any dimensions?
- How does the spider's height on the wall affect this problem? For which starting heights, h, would there be a different shortest route?
- Investigate the study of geodesics. How do geodesics apply to this problem?

A Follow Up Problem – The Russian Motorcyclists

The following is another problem from Henry Ernest Dudeney which was published in *The Strand Magazine*, Volume 53 (1917).

> Two Army motorcyclists, on the road at Adjbkmlprzll, wish to go to Brczrtwxy, which, for the sake of brevity, are marked in the accompanying map as A and B. Now, Pipipoff said: "I shall go to D, which is six miles, and then take the straight road to B, another fifteen miles." But Sliponsky thought he would try the upper road by way of C. Curiously enough, they found on reference to their cyclometers that the distance either way was exactly the same. This being so, they ought to have been able easily to answer the General's simple question, "How far is it from A to C?" It can be done in the head in a few moments, if you only know how. Can the reader state correctly the distance?



There are several ways to solve this problem with a bit of algebra and the application of the Pythagorean Theorem. Dudeney cryptically states that, "It can be done in the head in a few moments, if you only know how." Can you deduce the clever solution method that Dudeney is referring to?

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