CC335. Dans le triangle ABC, BD est la médiane du côté AC et DG est parallèle à la base BC (G est le point d'intersection de la droite parallèle avec le côté AB). Dans le triangle ABD, AE est la médiane du côté BD et F est le point d'intersection des segments DG et AE. Trouvez le rapport $\dfrac{BC}{FG}$.

CC336. Considérez le triangle ABC tel que $\angle B = \angle C = 70°$. Sur les côtés AB et AC, on prend les points F and E tels que $\angle ABE = 15°$ and $\angle ACF = 30°$. Trouvez $\angle AEF$.

CONTEST CORNER SOLUTIONS

Statements of the problems in this section originally appear in 2017: 43(7), p. 281–281.

CC281. In the Original Six era of the NHL, one particular season, each team played 20 games (each team played the other 5 teams 4 times each). Each game ended as a win, a loss or a tie (there were no 'overtime losses'). At the end of this certain season, the standings were as below. What were all the possible outcomes for Montreal's number of wins X, losses Y and ties Z?

Team	Wins	Losses	Ties
Toronto	2	12	6
Boston	6	10	4
Detroit	7	12	1
New York	7	9	4
Chicago	11	7	2
Montreal	x	y	z

Originally Question 6 from the 2015 W.J. Blundon Mathematics Contest.

We received 7 solutions, all of which were correct and complete. We present the solution by Fernando Ballesta Yagüe.

The known results are:

- $2 + 6 + 7 + 7 + 11 = 33$ wins.
- $12 + 10 + 12 + 9 + 7 = 50$ losses.
- $6 + 4 + 1 + 4 + 2 = 17$ ties.

As we eventually need to have the same total number of wins and losses (if a team loses a match, another team has had to win that match), we have that $x + 33$, which is the total number of wins, has to be equal to $y + 50$, which is the total number of losses. Since the total number of matches played by Montreal's team is 20, we have:

$$x + y + z = 20,$$
$$x + 33 = y + 50.$$

From that we can deduce that

$$x = y + 17 \to (y + 17) + y + z = 20 \to z = 3 - 2y.$$

Since the total number of ties, which is $z + 17$, has to be even (if a match has ended in a tie, it has ended in a tie for both of the teams), we have that $z > 0$ and z is odd. The only two possibilities for $z > 0$ are $y = 0$ and $y = 1$. If $y = 0$, then $z = 3$ and $x = 17$; if $y = 10$, then $z = 1$ and $x = 18$. So the possible outcomes for Montreal's number of wins, losses and ties are:

$$x = 17,\ y = 0,\ z = 3$$
$$x = 18,\ y = 1,\ z = 1$$

CC282. Calculate the value of

$$\left(3^{4/3} - 3^{1/3}\right)^3 + \left(3^{5/3} - 3^{2/3}\right)^3 + \left(3^{6/3} - 3^{3/3}\right)^3 + \cdots + \left(3^{2006/3} - 3^{2003/3}\right)^3.$$

Originally Question 7 from the 2015 W.J. Blundon Mathematics Contest.

We received 20 submissions of which 17 were correct and complete. We present two solutions.

Solution 1, by Ivko Dimitrić.

Note that the difference of the two exponents of 3 of the pair of terms within each set of parentheses is 1, which means that the first term in each pair is 3 times the second one, i.e. those expressions are of the form

$$3^{(k+3)/3} - 3^{k/3} = 3 \cdot 3^{k/3} - 3^{k/3} = 2 \cdot 3^{k/3} = 2 \cdot \sqrt[3]{3^k},$$

for $k = 1, 2, \ldots, 2003$. Then the sum equals

$$\begin{aligned}
&(2 \cdot \sqrt[3]{3})^3 + (2 \cdot \sqrt[3]{3^2})^3 + (2 \cdot \sqrt[3]{3^3})^3 + \cdots + (2 \cdot \sqrt[3]{3^{2003}})^3 \\
&= 2^3 \cdot 3 + 2^3 \cdot 3^2 + 2^3 \cdot 3^3 + \cdots + 2^3 \cdot 3^{2003} \\
&= 8 \cdot (3 + 3^2 + 3^3 + \cdots + 3^{2003}) \\
&= 8 \cdot \frac{3\,(3^{2003} - 1)}{2} \\
&= 12\,(3^{2003} - 1).
\end{aligned}$$

Solution 2, by Miguel Amengual Covas.

The answer is $12\left(3^{2003}-1\right)$.

We give a generalization which, in the case $n = 2003$, yields the solution to the proposed problem.

$$\begin{aligned}
&\left(3^{\frac{4}{3}}-3^{\frac{1}{3}}\right)^3+\left(3^{\frac{5}{3}}-3^{\frac{2}{3}}\right)^3+\left(3^{\frac{6}{3}}-3^{\frac{3}{3}}\right)^3+\cdots+\left(3^{\frac{n+3}{3}}-3^{\frac{n}{3}}\right)^3 \qquad (1)\\
&=\left[3^{\frac{1}{3}}(3-1)\right]^3+\left[3^{\frac{2}{3}}(3-1)\right]^3+\left[3^{\frac{3}{3}}(3-1)\right]^3+\cdots+\left[3^{\frac{n}{3}}(3-1)\right]^3\\
&=3\cdot 2^3+3^2\cdot 2^3+3^3\cdot 2^3\cdots+3^n\cdot 2^3\\
&=2^3\left(3+3^2+3^3+\cdots+3^n\right).
\end{aligned}$$

The sum $3+3^2+3^3+\cdots+3^n$ is a geometric progression with value

$$\frac{3^n\cdot 3-3}{3-1}=\frac{3\left(3^n-1\right)}{2}.$$

Substituting this value in (1), we get

$$12\left(3^{2003}-1\right).$$

CC283. Two bags, Bag A and Bag B, each contain 9 balls. The 9 balls in each bag are numbered from 1 to 9. Suppose one ball is removed randomly from Bag A and another ball from Bag B. If X is the sum of the numbers on the balls left in Bag A and Y is the sum of the numbers of the balls remaining in Bag B, what is the probability that X and Y differ by a multiple of 4?

Originally Question 10 from the 2015 W.J. Blundon Mathematics Contest.

We received eight submissions to this problem, all of which were correct. We present the solution by Steven Chow, Miguel Amengual Covas, and Ballesta Yagüe Fernando (all done independently), modified by the editor.

Let x, y be the number of the ball removed from the Bag A and B, respectively. We have that

$$|X-Y|=|\,(45-x)-(45-y)\,|=|-x+y|.$$

X and Y differ by a multiple of 4 if and only if x and y are congruent modulo 4. From 1 to 9, the number of integers congruent to 0, 1, 2, 3 modulo 4 are 2, 3, 2, 2, respectively, so the probability that the numbers on the balls removed are congruent modulo 4 is

$$\left(\frac{2}{9}\right)^2+\left(\frac{3}{9}\right)^2+\left(\frac{2}{9}\right)^2+\left(\frac{2}{9}\right)^2=\frac{7}{27}.$$

Therefore the probability that X and Y differ by a multiple of 4 is $\frac{7}{27}$.

CC284. Define the function $f(x)$ to be the largest integer less than or equal to x for any real x. For example, $f(1) = 1, f(3/2) = 1, f(7/2) = 3, f(7/3) = 2$. Let

$$g(x) = f(x) + f(x/2) + f(x/3) + \cdots + f(x/(x-1)) + f(x/x).$$

a) Calculate $g(4) - g(3)$ and $g(7) - g(6)$.

b) What is $g(116) - g(115)$?

Originally Question 10 from the 2016 W.J. Blundon Mathematics Contest.

We received 8 correct solutions. We present the solution by Titu Zvonaru.

For positive integers$n$, we denote by $d(n)$ the number of divisors of n. Let $k = 1, 2, ..., n$. Dividing n by k, we have $n = pk + r$, with $0 \leq r < k$.

If $0 < r$, then $n - 1 = pk + (r - 1)$. Hence, $f\left(\frac{n}{k}\right) = f\left(\frac{n-1}{k}\right)$.

If $r = 0$, then $n-1 = (p-1)k+(k-1)$. Hence, $f\left(\frac{n}{k}\right) - f\left(\frac{n-1}{k}\right) = p-(p-1) = 1$.

We deduce that

$$g(n) - g(n-1) = d(n),$$

Since $4 = 2^2$, $7 = 7$, and $116 = 2^2 \cdot 29$, we have

$$g(4) - g(3) = d(4) = 3$$
$$g(7) - g(6) = d(7) = 2$$
$$g(116) - g(115) = d(116) = 6.$$

CC285. Find all values of k so that $x^2 + y^2 = k^2$ will intersect the circle with equation $(x - 5)^2 + (y + 12)^2 = 49$ at exactly one point.

Originally Question 6 from the 2016 W.J. Blundon Mathematics Contest.

We received 15 solutions, of which 10 were correct and complete and 5 were incomplete. One of the correct solutions was in Spanish. We present here the solution by Dan Daniel.

The two circles intersect in one point (internally or externally) if either $O_1O_2 = r_1 + r_2$ or $O_1O_2 = \|r_1 - r_2\|$, where we define O_1 as (0,0) and O_2 as (5,12). We then have $O_1O_2 = \sqrt{(5-0)^2 + (-12-0)^2} = 13$. So $r_1 = \|k\|$, and $r_2 = 7$.

The first case gives

$$\|k\| + 7 = 13 \implies \|k\| = 6 \implies k = \pm 6,$$

while the second gives

$$\|\|k\| - 7\| = 13 \implies \|k\| - 7 = \pm 13 \implies \|k\| = 20 \implies k = \pm 20.$$

Therefore $k \in \{-20, -6, 6, 20\}$.