

# THE CONTEST CORNER

No. 67

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*The problems featured in this section have appeared in, or have been inspired by, a mathematics contest question at either the high school or the undergraduate level. Readers are invited to submit solutions, comments and generalizations to any problem. Please see submission guidelines inside the back cover or online.*

*To facilitate their consideration, solutions should be received by **February 1, 2019**.*

*The editor thanks Valérie Lapointe, Carignan, QC, for translations of the problems.*



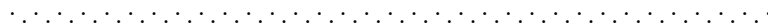
**CC331.** Consider triangle  $ABC$  with  $\angle B = \angle C = 70^\circ$ . On the sides  $AB$  and  $AC$ , we take the points  $F$  and  $E$ , respectively, so that  $\angle ABE = 15^\circ$  and  $\angle ACF = 30^\circ$ . Find  $\angle AEF$ .

**CC332.** Find the largest integer  $k$  such that  $135^k$  divides  $2016!$ . Note that  $n! = 1 \cdot 2 \cdot 3 \cdots n$ .

**CC333.** Let  $\theta = \arctan 2 + \arctan 3$ . Find  $\frac{1}{\sin^2 \theta}$  and simplify fully.

**CC334.** Find the sum of all positive integers  $x$  for which  $x + 56$  and  $x + 113$  are perfect squares.

**CC335.** In the triangle  $ABC$ ,  $BD$  is the median to the side  $AC$ ,  $DG$  is parallel to the base  $BC$  ( $G$  is the point of intersection of the parallel with  $AB$ ). In the triangle  $ABD$ ,  $AE$  is the median to the side  $BD$  and  $F$  is the intersection point of  $DG$  and  $AE$ . Find  $\frac{BC}{FG}$ .



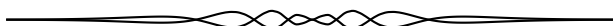
**CC331.** Considérez le triangle  $ABC$  tel que  $\angle B = \angle C = 70^\circ$ . Sur les côtés  $AB$  et  $AC$ , on prend les points  $F$  et  $E$  tels que  $\angle ABE = 15^\circ$  et  $\angle ACF = 30^\circ$ . Trouvez  $\angle AEF$ .

**CC332.** Trouvez le plus grand entier  $k$  tel que  $135^k$  divise  $2016!$ , où  $n! = 1 \cdot 2 \cdot 3 \cdots n$ .

**CC333.** Soit  $\theta = \arctan 2 + \arctan 3$ . Trouvez  $\frac{1}{\sin^2 \theta}$  et simplifiez le plus possible.

**CC334.** Trouvez la somme de tous les entiers positifs  $x$  tels que  $x + 56$  et  $x + 113$  sont des carrés parfaits.

**CC335.** Dans le triangle  $ABC$ ,  $BD$  est la médiane du côté  $AC$  et  $DG$  est parallèle à la base  $BC$  ( $G$  est le point d'intersection de la droite parallèle avec le côté  $AB$ ). Dans le triangle  $ABD$ ,  $AE$  est la médiane du côté  $BD$  et  $F$  est le point d'intersection des segments  $DG$  et  $AE$ . Trouvez le rapport  $\frac{BC}{FG}$ .



## CONTEST CORNER SOLUTIONS

*Statements of the problems in this section originally appear in 2017: 43(7), p. 281–281.*



**CC281.** In the Original Six era of the NHL, one particular season, each team played 20 games (each team played the other 5 teams 4 times each). Each game ended as a win, a loss or a tie (there were no ‘overtime losses’). At the end of this certain season, the standings were as below. What were all the possible outcomes for Montreal’s number of wins  $X$ , losses  $Y$  and ties  $Z$ ?

Team	Wins	Losses	Ties
Toronto	2	12	6
Boston	6	10	4
Detroit	7	12	1
New York	7	9	4
Chicago	11	7	2
Montreal	$x$	$y$	$z$

*Originally Question 6 from the 2015 W.J. Blundon Mathematics Contest.*

*We received 8 solutions, all of which were correct and complete. We present the solution by Fernando Ballesta Yagüe.*

The known results are:

- $2 + 6 + 7 + 7 + 11 = 33$  wins.
- $12 + 10 + 12 + 9 + 7 = 50$  losses.
- $6 + 4 + 1 + 4 + 2 = 17$  ties.