THE CONTEST CORNER

No. 59

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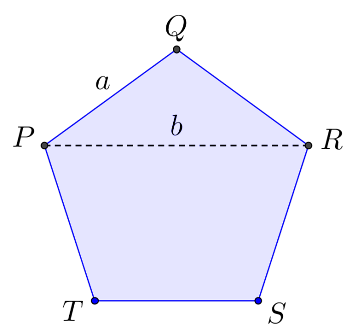
The problems featured in this section have appeared in, or have been inspired by, a mathematics contest question at either the high school or the undergraduate level. Readers are invited to submit solutions, comments and generalizations to any problem. Please see submission guidelines inside the back cover or online.

To facilitate their consideration, solutions should be received by **May 1, 2018**.

The editor thanks André Ladouceur, Ottawa, ON, for translations of the problems.

CC291. The point P is on the parabola $x^2 = 4y$. The tangent at P meets the line $y = -1$ at the point A. For the point $F(0,1)$, prove that $\angle AFP = 90^\circ$ for all positions of P, except $(0,0)$.

CC292. Let a be the length of a side and b be the length of a diagonal in the regular pentagon $PQRST$ as shown.



Prove that

$$\frac{b}{a} - \frac{a}{b} = 1.$$

CC293. The transformation $T : (x,y) \mapsto \left(-\frac{1}{2}(3x-y), -\frac{1}{2}(x+y)\right)$ is applied repeatedly to the point $P_0(3,1)$, which produces a sequence of points $P_1, P_2, \ldots$. Show that the area of the convex quadrilateral defined by any four consecutive points is constant.

CC294.

a) Prove that $\sin 2A = \dfrac{2\tan A}{1+\tan 2A}$, where $0 < A < \pi/2$.

b) If $\sin 2A = 4/5$, find $\tan A$.

CC295. In how many ways is it possible to choose four distinct integers from $1, 2, 3, 4, 5, 6$ and 7, so that their sum is even?

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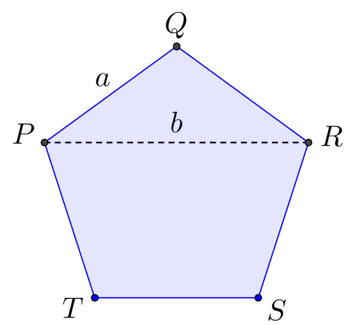
CC291. On considère un point P sur la parabole d'équation $x^2 = 4y$. La tangente au point P coupe la droite d'équation $y = -1$ au point A. Soit le point $F(0, 1)$. Démontrer que $\angle AFP = 90°$ pour toutes les positions de P, à l'exception de $(0, 0)$.

CC292. Dans la figure suivante, $PQRST$ est un pentagone régulier, a est la longueur de ses côtés et b est la longueur de ses diagonales.



Démontrer que

$$\frac{b}{a} - \frac{a}{b} = 1.$$

CC293. On fait subir la transformation $T : (x, y) \mapsto \left(-\frac{1}{2}(3x - y), -\frac{1}{2}(x + y)\right)$ de façon répétée au point $P_0(3, 1)$ et aux images successives, ce qui produit une suite de points $P_1, P_2, \ldots$ Démontrer que l'aire du quadrilatère convexe défini par n'importe quels quatre points consécutifs est constante.

CC294.

a) Démontrer que $\sin 2A = \dfrac{2\tan A}{1 + \tan 2A}$, où $0 < A < \pi/2$.

b) Sachant que $\sin 2A = \dfrac{4}{5}$, déterminer $\tan A$.

CC295. De combien de façons est-il possible de choisir quatre entiers distincts, parmi les entiers $1, 2, 3, 4, 5, 6$ et 7, de manière que leur somme soit paire?