# PROBLEMS

*Readers are invited to submit solutions, comments and generalizations to any problem in this section. Moreover, readers are encouraged to submit problem proposals. Please see submission guidelines inside the back cover or online.*

*To facilitate their consideration, solutions should be received by* **March 1, 2018**.

*The editor thanks Rolland Gaudet, retired professor of Université de Saint-Boniface in Winnipeg, for translations of the problems.*

*An asterisk* ($\star$) *after a number indicates that a problem was proposed without a solution.*

**4261**. *Proposed by Margarita Maksakova.*

Consider the chess board. A baron can move only on the black squares and in one move he can go from one black square to any of the diagonally adjacent black squares. What is the smallest number of moves he needs to go to every black square?

**4262**. *Proposed by Prithwijit De.*

Let $a_1, a_2, \ldots, a_n$ be positive integers and suppose $\sum_{k=1}^{n} a_k = S$. Find the smallest positive value of $c$ such that the equation

$$\sum_{k=1}^{n} \frac{a_k x^k}{1+x^{2k}} = c$$

has a unique real solution.

**4263**. *Proposed by Michel Bataille.*

Let $ABC$ be a triangle. Let $\Gamma$, with centre $O$ and radius $R$, be the circumcircle of $ABC$ and $\gamma$, with centre $I \neq O$ and radius $r$, be the incircle of $ABC$. Let $D, E, F$ be the orthogonal projections of the inverse of $I$ in $\Gamma$ onto $BC, CA, AB$, respectively. Express the circumradius of $\Delta DEF$ as a function of $R$ and $r$.

**4264**. *Proposed by Dorin Marghidanu and Leonard Giugiuc.*

Let $(a_n)$ and $(b_n)$ be two sequences such that $a_0, b_0 > 0$ and

$$a_{n+1} = a_n + \frac{1}{2b_n} \quad \text{and} \quad b_{n+1} = b_n + \frac{1}{2a_n}$$

for all $n \geq 0$. Prove that

$$\max(a_{2017}, b_{2017}) > 44.$$

**4265**. *Proposed by Daniel Sitaru.*

Consider real numbers $a, b, c \in (0,1)$ such that $a+b+c=1$. Show that

$$\frac{4}{\pi}(\arctan a + \arctan b + \arctan c) > \frac{1}{2-(ab+bc+ca)}.$$

**4266**. *Proposed by Marius Stănean.*

Let $ABC$ be a triangle with orthocenter $H$. Let $HM$ be the median and $HS$ be the symmedian in triangle $BHC$. Denote by $P$ the orthogonal projection of $A$ onto $HS$. Prove that the circumcircle of triangle $MPS$ is tangent to the circumcircle of triangle $ABC$.

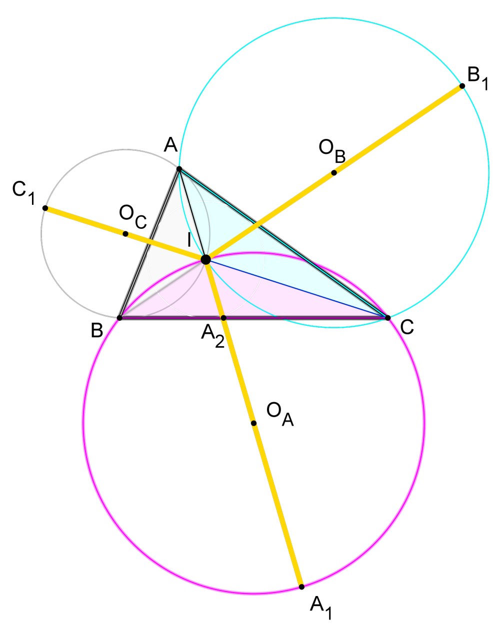
**4267**. *Proposed by Leonard Giugiuc.*

Let $a, b, c$ and $d$ be real numbers such that $0 < a, b, c \leq 1$ and $abcd = 1$. Prove that

$$5(a+b+c+d) + \frac{4}{abc+abd+acd+bcd} \geq 21.$$

**4268**. *Proposed by Mihaela Berindeanu.*

Let $I$ be the incenter of the acute triangle $ABC$, and let the triangle's internal angle bisectors intersect the circles $IBC$, $ICA$, and $IAB$ again at $A_1$, $B_1$, and $C_1$, respectively. Show that $\overrightarrow{IA_1} + \overrightarrow{IB_1} + \overrightarrow{IC_1} = \overrightarrow{0}$ if and only if $\triangle\ ABC$ is equilateral.



**4269**. *Proposed by Hung Nguyen Viet.*

Let $x_1, x_2, \ldots, x_n$ be real numbers such that

$$\sin x_1 \cos x_2 + \sin x_2 \cos x_3 + \cdots + \sin x_n \cos x_1 = \frac{n}{2}.$$

Prove that

$$\cos 2x_1 + \cos 2x_2 + \cdots + \cos 2x_n = 0.$$

**4270**. *Proposed by Leonard Giugiuc.*

Let $k$ and $t$ be real numbers with $k \in (0,1)$ and $t \in [\frac{\pi}{4}, \frac{\pi}{2}]$. Prove that

$$\int_0^t \frac{\cos x}{x^k} dx \geq \int_0^t \frac{\sin x}{x^k} dx.$$

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**4261**. *Proposé par Margarita Maksakova.*

Soit un échiquier. Bernadette y déplace un jeton, allant d'un carré noir à un de ses carrés noirs diagonalement adjacents. Quel est le plus petit nombre de tels déplacements qui permettra de visiter tous les carrés noirs?

**4262**. *Proposé par Prithwijit De.*

Soient $a_1, a_2, \ldots, a_n$ des entiers positifs tels que $\sum_{k=1}^{n} a_k = S$. Déterminer la plus petite valeur positive de $c$ telle que l'équation

$$\sum_{k=1}^{n} \frac{a_k x^k}{1 + x^{2k}} = c$$

possède une solution réelle unique.

**4263**. *Proposé par Michel Bataille.*

Soit $ABC$ un triangle avec cercle circonscrit $\Gamma$ de centre $O$ et rayon $R$, puis cercle inscrit $\gamma$ de centre $I \neq O$ et rayon $r$. Soient $D, E$ et $F$ les projections orthogonales de l'inverse de $I$ dans $\Gamma$ vers $BC, CA$ et $AB$ respectivement. Exprimer le rayon du cercle circonscrit de $\Delta DEF$ en termes de $R$ et $r$.

**4264**. *Proposé par Dorin Marghidanu et Leonard Giugiuc.*

Soient $(a_n)$ et $(b_n)$ deux suites telles que $a_0, b_0 > 0$ puis

$$a_{n+1} = a_n + \frac{1}{2b_n} \quad \text{et} \quad b_{n+1} = b_n + \frac{1}{2a_n}$$

pour tout $n \geq 0$. Démontrer que

$$\max(a_{2017}, b_{2017}) > 44.$$

**4265**. *Proposé par Daniel Sitaru.*

Soient des nombres réels $a, b, c \in (0, 1)$ tels que $a + b + c = 1$. Démontrer que

$$\frac{4}{\pi}(\arctan a + \arctan b + \arctan c) > \frac{1}{2 - (ab + bc + ca)}.$$

**4266**. *Proposé par Marius Stănean.*

Soit $ABC$ un triangle avec orthocentre $H$. Soient $HM$ la médiane et $HS$ la symédiane dans le triangle $BHC$. Dénoter par $P$ la projection orthogonale de $A$ vers $HS$. Démontrer que le cercle circonscrit du triangle $MPS$ est tangent au cercle circonscrit du triangle $ABC$.

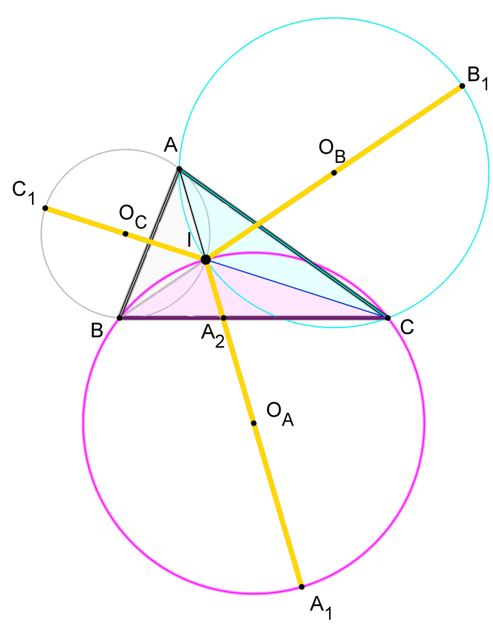
**4267**. *Proposé par Leonard Giugiuc.*

Soient $a, b, c$ et $d$ des nombres réels tels que $0 < a, b, c \leq 1$ and $abcd = 1$. Démontrer que

$$5(a + b + c + d) + \frac{4}{abc + abd + acd + bcd} \geq 21.$$

**4268**. *Proposé par Mihaela Berindeanu.*

Soit $I$ le centre du cercle inscrit du triangle aigu $ABC$ et supposer que ses bissectrices internes d'angles intersectent les cercles $IBC, ICA$ et $IAB$ une seconde fois en $A_1, B_1$ et $C_1$ respectivement. Démontrer que $\overrightarrow{IA_1} + \overrightarrow{IB_1} + \overrightarrow{IC_1} = \overrightarrow{0}$ si et seulement si $\triangle\, ABC$ est équilatéral.



**4270**. *Proposé par Hung Nguyen Viet.*

Soient $x_1, x_2, \ldots, x_n$ des nombres réels tels que

$$\sin x_1 \cos x_2 + \sin x_2 \cos x_3 + \cdots + \sin x_n \cos x_1 = \frac{n}{2}.$$

Démontrer que

$$\cos 2x_1 + \cos 2x_2 + \cdots + \cos 2x_n = 0.$$

**4270**. *Proposé par Leonard Giugiuc.*

Soient $k$ et $t$ des nombres réels tels que $k \in (0,1)$ et $t \in [\frac{\pi}{4}, \frac{\pi}{2}]$. Démontrer que

$$\int_0^t \frac{\cos x}{x^k} dx \geq \int_0^t \frac{\sin x}{x^k} dx.$$

## Math Quotes

I wanted certainty in the kind of way in which people want religious faith. I thought that certainty is more likely to be found in mathematics than elsewhere. But I discovered that many mathematical demonstrations, which my teachers expected me to accept, were full of fallacies, and that, if certainty were indeed discoverable in mathematics, it would be in a new field of mathematics, with more solid foundations than those that had hitherto been thought secure. But as the work proceeded, I was continually reminded of the fable about the elephant and the tortoise. having constructed an elephant upon which the mathematical world could rest, I found the elephant tottering, and proceeded to construct a tortoise to keep the elephant from falling. But the tortoise was no more secure than the elephant, and after some twenty years of very arduous toil, I came to the conclusion that there was nothing more that I could do in the way of making mathematical knowledge indubitable.

*Bertrand Russell in "Portraits from Memory."*