Selected Problems from the Early Years of the Moscow Mathematical Olympiad

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I am a trainer of the Malaysian National Team which competes in the International Mathematical Olympiad. I was a team member myself from 2006 to 2009, and was the first Malaysian to win a silver medal. I attended the 2016 IMO in Hong Kong as a Leader Observer.

In assembling training material, I turned to a place with a rich mathematical tradition, the former Soviet Union. I came across a file containing problems from this contest between 1935 and 1941 inclusive. The papers were incomplete as parts had been lost due to passage of time, so I treated them as isolated problems. The content, style and level of difficulty were certainly quite different from the IMO problems nowadays. Nevertheless, I found some of them useful for my purpose.

Below is a sample of ten Moscow Mathematical Olympiad problems. The solutions will be given in the next issue.

1. Solve the system of equations $x^2 + y^2 - 2z^2 = 2a^2$, $x + y + 2z = 4(a^2 + 1)$ and $z^2 - xy = a^2$ where a is a real constant. (1935)

2. Which is larger, 300! or 100^{300}? (1940)

3. (a) Find all possible values of a such that for all x and some integers b and c, $(x - a)(x - 10) + 1 = (x - b)(x - c)$.

   (b) Find all possible triples (a, b, c) of distinct non-zero integers such that $x(x-a)(x-b)(x-c)+1$ is the product of two non-constant polynomials with integer coefficients. (1941)

4. How many planes are equidistant from four given points not all in a plane? (1938)

5. Given a line and a circle, construct a unit circle tangent to both. How many solutions are there? (1940)

6. O is the circumcentre of triangle ABC. P, Q and R are the points symmetric to O about BC, CA and AB respectively. Construct ABC given only the points P, Q and R. (1940)

7. Construct a triangle given the points of intersection of its circumcircle with the extensions of the altitude, angle bisector and median from the same vertex. (1935)

8. Given two points A and B not on a line ℓ, construct a point P on ℓ such that $AP + BP = 1$. (1937)

9. Given three non-collinear points, construct three circles, each passing through two of them, such that every two circles intersect, and the tangents to the circles at each point of intersection are perpendicular to each other. (1937)

10. When an infinite circular cone is cut along a line through its vertex, its surface opens up into a circular sector. A straight line ℓ is drawn perpendicular to the bisector of the central angle which has measure θ. When the cone is reconstructed, determine the number of points of self-intersection of ℓ in terms of θ. (1940)

Historical Remarks on the first Moscow Mathematical Olympiad

In the spring of 1935, the Board of the Moscow Mathematical Society, following the example of Leningrad, decided to organize the first Moscow Mathematical Olympiad. The organizing committee included all professors of mathematics from Moscow University and was headed by P. S. Alexandrov, who was then the President of the Moscow Mathematical Society. The purpose of the Olympiad was to find the most talented students, to attract attention of the young people at large to some of the most important problems and methods of modern mathematics, and to show the students, at least partly, what Soviet mathematicians are working on, what progress they have made and what challenges they have.

In the preliminary round, 314 high school students participated, and 120 of them made it into the final round. Three students were awarded first prizes, five got second prizes and, in addition, 44 students received honorable mentions. A place at the top of the Olympiad determined for many their future scientific career. By the initiative of the famous mathematician A. N. Kolmogorov, the problems in the final round focused on the following three different mathematical abilities:

geometric,

computational and algorithmic,

combinatorial and logical.

The Moscow Mathematical Olympiad has been held every year since 1935 except for the war years 1942 to 1944.

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