

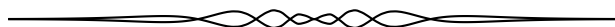
# THE CONTEST CORNER

No. 33

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*The problems featured in this section have appeared in, or have been inspired by, a mathematics contest question at either the high school or the undergraduate level. Readers are invited to submit solutions, comments and generalizations to any problem. Please see submission guidelines inside the back cover or online.*

*To facilitate their consideration, solutions should be received by the editor by **May 1, 2016**, although late solutions will also be considered until a solution is published.*

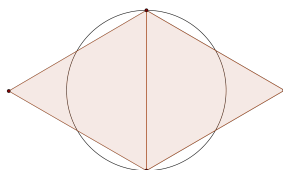


**CC161.** A number  $n$  written in base  $b$  reads 211, but it becomes 110 when written in base  $b + 2$ . Find  $n$  and  $b$  in base 10.

**CC162.** What is the probability that 99 divides a randomly chosen 4-digit palindrome?

**CC163.** If  $x$  is randomly chosen in  $[-100, 100]$ , what is the probability that  $g[f(x)]$  is negative given that  $f(x) = x^2 + 3x - 7$  and  $g(x) = x^2 - 2x - 99$ ?

**CC164.** Build two equilateral triangles on the diameter of a circle with radius 5. What is the total area of the circle outside the equilateral triangles?



**CC165.** Georges pays \$50 on each of four gas refills but the prices per litre were \$1.32, \$1.25, \$1.11 and \$1.18 as the price was fluctuating a lot in that time period. What is the average price per litre?

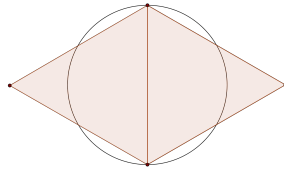


**CC161.** Un nombre  $n$  écrit en base  $b$  se lit 211, mais devient 110 quand on l'écrit dans la base  $b + 2$ . Trouver  $n$  et  $b$  en base 10.

**CC162.** Quelle est la probabilité que 99 divise un palindrome à 4 chiffres choisi au hasard?

**CC163.** Si on choisit  $x$  aléatoirement dans  $[-100, 100]$ , quelle est la probabilité que  $g[f(x)]$  est négatif compte tenu que  $f(x) = x^2 + 3x - 7$  et  $g(x) = x^2 - 2x - 99$ ?

**CC164.** Sur le diamètre d'un cercle de rayon 5, on construit 2 triangles équilatéraux. Quelle est l'aire totale du cercle en dehors des triangles équilatéraux?



**CC165.** Georges paye 50\$ à chacune de quatre visites à une station-service lorsque les prix par litre étaient 1,32\$, 1,25\$, 1,11\$ et 1,18\$ pendant une période de grande fluctuation de prix. Quel était le prix moyen par litre?



## CONTEST CORNER SOLUTIONS

*Statements of the problems in this section originally appear in 2014: 40(3), p. 96–97.*



**CC111.** Find all positive integers with two or more digits such that if we insert a 0 between the units and tens digits we get a multiple of the original number.

*Originally problem A1 from the 2003 Mexican Math Olympiad.*

*We received ten correct submissions. We present the solution by Yihang Dong.*

Call the number  $\overline{xy}$ , where  $y$  is the one's digit and  $x$  is the rest. Then we are looking for numbers such that  $k(10x + y) = 100x + y$ . If  $k > 10$ , then  $10kx > 100x$  and  $ky > y$ , which cannot happen. So  $k \leq 10$ .

Considering mod 10, we see that  $ky \equiv y \pmod{10}$ . So  $10a = ky - y$  for some  $a$ . One possibility is  $y = 0$ . If  $y \neq 0$ , then  $(k - 1)y = 10a$ . So  $10 \mid (k - 1)y$ . If  $5 \mid y$ , then  $y = 5$ . If  $5 \mid (k - 1)$ , then  $k - 1 = 5$  and so  $k = 6$ .

Let us go through these three possibilities.