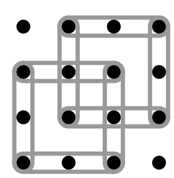
MAYHEM SOLUTIONS

Mathematical Mayhem *is being reformatted as a stand-alone mathematics journal for high school students. Solutions to problems that appeared in the last volume of* **Crux** *will appear in this volume, after which time* **Mathematical Mayhem** *will be discontinued in* **Crux**. *New* **Mayhem** *problems will appear when the journal is relaunched in 2013.*

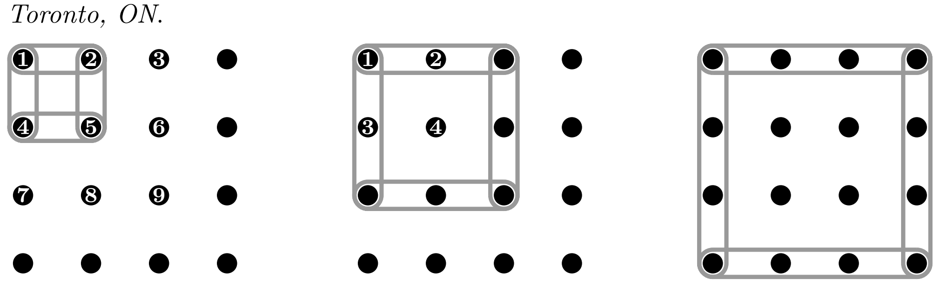
Due to a filing error the following solutions were received, but never acknowledged: DANIEL LOPEZ AGUAYO, Institute of Mathematics, UNAM, Morelia, Mexico (M489, M491, M493); AGAUSILIA DINDA ASMARA, student, SMPN 8, Yogyakarta, Indonesia (M482); GHINA ZHAFIRA ASTRIDIANTI, student, SMPN 8, Yogyakarta, Indonesia (M482); FLORENCIO CANO VARGAS, Inca, Spain(M481); ANDHIKA GILANG, student, SMPN 8, Yogyakarta, Indonesia (M482, M483, M484); GUSTAVO KRIMKER, Universidad CAECE, Buenos Aires, Argentina(M500); DAVID E. MANES, SUNY at Oneonta, Oneonta, NY, USA (M482, M484, M485, M486, M487); MUHAMMAD ROIHAN MUNAJIH, SMA Semesta Bilingual Boarding School, Indonesia(M483, M484); CARLOS TORRES NINAHUANCA, Lima, Perú(M484); AARON PERKINS and ADRIENNA BINGHAM, students, Angelo State University, San Angelo, TX, USA(M483, M484, M486); LAURENTIA ROSA RENATA, student, SMPN 8 ,Yogyakarta, Indonesia (M482); and MIHAÏ STOËNESCU, Bischwiller, France(M470, M471, M472, M473, M474, M475, M495, M497, M498, M499). The editor apologizes for the oversight.

M501. *Proposed by the Mayhem Staff.*

A 4 by 4 square grid is formed by removable pegs that are one centimetre apart as shown in the diagram. Elastic bands may be attached to pegs to form squares, two different 2 by 2 squares are shown in the diagram. How many different squares are possible?

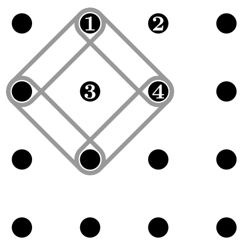


Solution by Gloria (Yuliang) Fang, University of Toronto Schools, Toronto, ON.

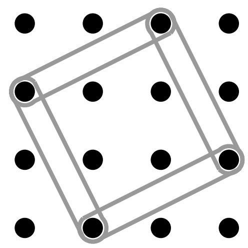


Type 1 **Type 2** **Type 3**

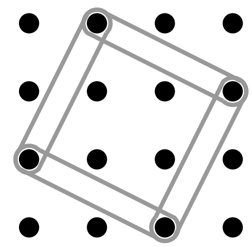
We can classify the types of solution, there are $3 \times 3 = 9$ squares of type 1, $2 \times 2 = 4$ squares of type 2, one square of type 3, $2 \times 2 = 4$ squares of type 4 and 2 squares of type 5 (labeled 5a and 5 b). In the diagrams with numbers, the numbers indicate the top or top left vertex for the other squares in that case. Thus there are $9 + 4 + 1 + 4 + 2 = 20$ possible squares.



Type 1



Type 5a



Type 5b

Also solved by FLORENCIO CANO VARGAS, Inca, Spain; GESINE GEUPEL, student, Max Ernst Gymnasium, Brühl, NRW, Germany; RICHARD I. HESS, Rancho Palos Verdes, CA, USA; MUHAMMAD ROIHAN MUNAJIH, SMA Semesta Bilingual Boarding School, Indonesia; RICARD PEIRÓ, IES "Abastos", Valencia, Spain; BRUNO SALGUEIRO FANEGO, Viveiro, Spain; CÁSSIO DOS SANTOS SOUSA, Instituto Tecnológico de Aeronáutica, São Paulo, Brazil; and MIHAÏ STOËNESCU, Bischwiller, France. Two incorrect solutions were received.

Salgueiro Fanego also pointed out that if the 4×4 grid was replaced by a $n \times n$ grid, there would be $\dfrac{n^4 - n^2}{12}$ possible squares. He points to the web page `http://www.arrakis.es/ mcj/prb141.htm` *where two solutions (in Spanish) are given.*

M502. *Proposed by the Mayhem Staff.*

At their last basketball game Alice, Bob and Cindy scored a total of 23 points between them. Each player got at least 1 point, and Cindy scored at least 10. How many different ways could the 23 points been awarded to satisfy the conditions? For example: 5 points for Alice, 3 points for Bob, 15 for Cindy; and 3 points for Alice, 5 points for Bob, 15 for Cindy; are two different possibilities.

Solution by Bruno Salgueiro Fanego, Viveiro, Spain.

There are a number of cases.

- If $c = 10$ then there are 12 possibilities given by $(a, b) = (a, 13 - a)$ with $1 \le a \le 12$.
- If $c = 11$ then there are 11 possibilities given by $(a, b) = (a, 12 - a)$ with $1 \le a \le 11$.
- If $c = 12$ then there are 10 possibilities given by $(a, b) = (a, 11 - a)$ with $1 \le a \le 10$.

$$\vdots$$

- If $c = 20$ then there are 2 possibilities given by $(a, b) = (a, 3 - a)$ with $1 \le a \le 2$.

- If $c = 21$ then there is only 1 possibility, $(a, b) = (1, 1)$.

It total, there are $12 + 11 + 10 + \cdots + 2 + 1 = 78$ possibilities.

Also solved by GEORGE APOSTOLOPOULOS, Messolonghi, Greece; FLORENCIO CANO VARGAS, Inca, Spain; GLORIA (YULIANG) FANG, University of Toronto Schools, Toronto, ON; GESINE GEUPEL, student, Max Ernst Gymnasium, Brühl, NRW, Germany; RICHARD I. HESS, Rancho Palos Verdes, CA, USA; MARDATILLA NUR JUWITA, student, SMP N 8 YOGYAKARTA, Indonesia; CARL LIBIS, Department of Mathematics, Community College of Rhode Island, Warwick, RI, USA; MUHAMMAD ROIHAN MUNAJIH, SMA Semesta Bilingual Boarding School, Indonesia; MIHAÏ STOËNESCU, Bischwiller, France; and KONSTANTINE ZELATOR, University of Pittsburgh, Pittsburgh, PA, USA. Three incorrect solutions were received.

M503. *Proposed by John Grant McLoughlin, University of New Brunswick, Fredericton, NB.*

Write any number and then follow that number by adjoining its reversal. For example, if you write 13 then you would get 1331. Show that the resulting number is always divisible by 11.

Solution by Konstantine Zelator, University of Pittsburgh, Pittsburgh, PA, USA.

Let $N = d_{n-1} \cdot 10^{n-1} + d_{n-2} \cdot 10^{n-2} + \cdots + d_1 \cdot 10 + d_0$ be an n digit number, where $0 \le d_i \le 9$ for each $i = 0, 1, \ldots, n-1$ are the numbers digits. Let M be the number created by adjoining the reversal of N to N, that is

$$\begin{aligned} M &= d_{n-1} \cdot 10^{2n-1} + d_{n-2} \cdot 10^{2n-2} + \cdots + d_1 \cdot 10^{n+1} + d_0 \cdot 10^n \\ &\qquad + d_0 \cdot 10^{n-1} + d_1 \cdot 10^{n-2} + \cdots + d_{n-2} \cdot 10 + d_{n-1} \\ &= d_{n-1}(10^{2n-1} + 1) + d_{n-2}(10^{2n-2} + 10) + \\ &\qquad \cdots + d_1(10^{n+1} + 10^{n-2}) + d_0(10^n + 10^{n-1}) \\ &= \sum_{i=1}^{n} d_{n-i}(10^{2n-i} + 10^{i-1}) \\ &= \sum_{i=1}^{n} d_{n-i} 10^{i-1}(10^{2n-2i+1} + 1) \end{aligned}$$

Now, $10 \equiv -1 \pmod{11}$, so

$$\begin{aligned} M &\equiv \sum_{i=1}^{n} d_{n-i}(-1)^{i-1}((-1)^{2n-2i+1} + 1) \\ &\equiv \sum_{i=1}^{n} d_{n-i}(-1)^{i-1}([(-1)^{n-i}]^2(-1) + 1) \\ &\equiv \sum_{i=1}^{n} d_{n-i}(-1)^{i-1}((1)(-1) + 1) \\ &\equiv 0 \pmod{11}, \end{aligned}$$

completing the proof that when a number is created as in the statement of the problem, it is always divisible by 11.

Also solved by GEORGE APOSTOLOPOULOS, Messolonghi, Greece; FLORENCIO CANO VARGAS, Inca, Spain; IOAN VIOREL CODREANU, Secondary School, Satulung, Maramureş, Romania; GLORIA (YULIANG) FANG, University of Toronto Schools, Toronto, ON; GESINE GEUPEL, student, Max Ernst Gymnasium, Brühl, NRW, Germany; RICHARD I. HESS, Rancho Palos Verdes, CA, USA; CARL LIBIS, Department of Mathematics, Community College of Rhode Island, Warwick, RI, USA; DAVID E. MANES, SUNY at Oneonta, Oneonta, NY, USA; RICARD PEIRÓ, IES "Abastos", Valencia, Spain; CAO MINH QUANG, Nguyen Binh Khiem High School, Vinh Long, Vietnam; HENRY RICARDO, Tappan, NY, USA; BRUNO SALGUEIRO FANEGO, Viveiro, Spain; CÁSSIO DOS SANTOS SOUSA, Instituto Tecnológico de Aeronáutica, São Paulo, Brazil; MIHAÏ STOËNESCU, Bischwiller, France; and the proposer.

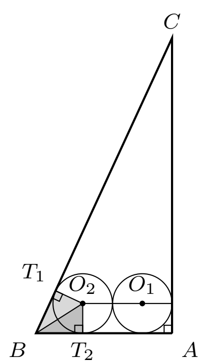
Several solvers used the divisibility by 11 *criterion, that is, a number is divisible by* 11 *if the alternating sum of its digits is divisible by* 11*. So, for example, when considering* 1331*, we would look at the sum* $1-3+3-1=0$ *which is divisible by* 11*, thus so is* 1331*. Salgueiro Fanego pointed out that the problem is a generalization of problem* **M297** *[2007 : 201, 202; 2008 : 205].*

M504. *Proposed by Bruce Shawyer, Memorial University of Newfoundland, St. John's, NL.*

Inside a right triangle with sides 3, 4, 5, two equal circles are drawn that are tangent to one another and to one leg. One circle of the pair is tangent to the hypotenuse. The other circle of the pair is tangent to the other leg. Determine the radii of the circles in both cases.

Solution by Richard I. Hess, Rancho Palos Verdes, CA, USA.

Consider the general case shown for arbitrary angle $\angle ABC = \theta$. Let the circles' radii be r. As the circle with centre O_2 is tangent to both AB and BC, radii to the points of tangency, T_1 and T_2, are perpendicular to the sides. Thus, as right triangle ΔO_2BT_1 and ΔO_2BT_2 share their hypotenuse and have another equal side, which are radii of the circle with centre O_2, then they are congruent, hence $\angle O_2BT_2 = \angle O_2BT_2 = \frac{1}{2}\angle ABC = \frac{\theta}{2}$. Hence, we see that $BT_2 = r\cot\frac{\theta}{2}$, then



$$AB = 3r + r\cot\frac{\theta}{2} = r\left(3+\cot\frac{\theta}{2}\right) \Rightarrow r = \frac{AB}{3+\cot\frac{\theta}{2}}. \qquad (1)$$

Using the identity $\cot\frac{\theta}{2} = \frac{1+\cos\theta}{\sin\theta}$, (1) can be rewritten

$$r = \frac{AB\sin\theta}{3\sin\theta+\cos\theta+1}. \qquad (2)$$

In the problem posed, the first position has $AB = 3$, $\sin\theta = \frac{4}{5}$ and $\cos\theta = \frac{3}{5}$. Using (2) we get $r = \frac{3}{5}$.

Similarly, in the second position we have $AB = 4$, $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$. Using (2) we get $r = \frac{2}{3}$.

Also solved by FLORENCIO CANO VARGAS, Inca, Spain; GESINE GEUPEL, student, Max Ernst Gymnasium, Brühl, NRW, Germany; RICARD PEIRÓ, IES "Abastos", Valencia, Spain; BRUNO SALGUEIRO FANEGO, Viveiro, Spain; MIHAÏ STOËNESCU, Bischwiller, France; and the proposer. One incomplete solution and one incorrect solution were received.

M505. *Proposed by Neculai Stanciu, George Emil Palade Secondary School, Buzău, Romania.*

Prove that, for all positive integers n, the quantities $A = 5n + 7$ and $B = 6n^2 + 17n + 12$ are coprime (i.e. have no common factors other than 1).

Solution by Ioan Viorel Codreanu, Secondary School, Satulung, Maramureş, Romania.

Let $d = \gcd(A, B)$. Then

$$d \mid \left[5(6n^2 + 17n + 12) - 6n(5n + 7)\right]$$

namely $d \mid 43n + 60$.

By $d \mid 5n + 7$ and $d \mid 43n + 60$ we deduce that

$$d \mid [43(5n + 7) - 5(43n + 60)]$$

namely $d \mid 1$, hence $d = 1$ which concludes the solution.

Also solved by MIGUEL AMENGUAL COVAS, Cala Figuera, Mallorca, Spain; GEORGE APOSTOLOPOULOS, Messolonghi, Greece; FLORENCIO CANO VARGAS, Inca, Spain; GLORIA (YULIANG) FANG, University of Toronto Schools, Toronto, ON; RICHARD I. HESS, Rancho Palos Verdes, CA, USA; DAVID E. MANES, SUNY at Oneonta, Oneonta, NY, USA; MITEA MARIANA, No. 2 Secondary School, Cugir, Romania; MUHAMMAD ROIHAN MUNAJIH, SMA Semesta Bilingual Boarding School, Indonesia; RICARD PEIRÓ, IES "Abastos", Valencia, Spain; CAO MINH QUANG, Nguyen Binh Khiem High School, Vinh Long, Vietnam; BRUNO SALGUEIRO FANEGO, Viveiro, Spain; MIHAÏ STOËNESCU, Bischwiller, France; KONSTANTINE ZELATOR, University of Pittsburgh, Pittsburgh, PA, USA; and the proposer.

M506. *Proposed by John Grant McLoughlin, University of New Brunswick, Fredericton, NB; and Bruce Shawyer, Memorial University of Newfoundland, St. John's, NL.*

We are trying to create a set of positive integers, that each can be formed using their own digits only, along with any mathematical operations and/or symbols that are familiar to you. Each expression must in3ude at least one symbol/operation; the number of times a digit appears is the same as in the number itself. For example, $1 = \sqrt{1}$, $36 = 6 \times 3!$ and $121 = 11^2$. All valid contributions will be acknowledged.

Solutions by Gesine Geupel, student, Max Ernst Gymnasium, Brühl, NRW, Germany(1); Richard I. Hess, Rancho Palos Verdes, CA, USA(2); Carl Libis, Cumberland University, Lebanon, TN, USA(3); Ricard Peiró, IES "Abastos", Valencia, Spain(4); Bruno Salgueiro Fanego, Viveiro, Spain(5); Cássio dos Santos Sousa, Instituto Tecnológico de Aeronáutica, São Paulo, Brazil(6); Titu Zvonaru, Comăneşti, Romania(7); UNB-ESSO-CMS Math Camp 2012 (8); and the proposers (9).

$1 = \sqrt{1}$	8,9
$1 = 1!$	2, 6,8
$2 = 2!$	1, 2, 6,8
$2 = (2!)! = ((2!)!)! = \cdots$	6
$2 = -\lfloor -\sqrt{2} \rfloor$	7
$3 = -\lfloor -\sqrt{3!} \rfloor$	7
$3 = \left\lceil \sqrt{\sqrt{\sqrt{(3!)!}}} \right\rceil$	2
$3 = \left\lceil \sqrt{3!} \right\rceil$	8
$4 = \lfloor \sqrt{4!} \rfloor$	2, 7
$5 = \left\lceil \sqrt{\left\lceil \sqrt{\sqrt{5!}} \right\rceil !} \right\rceil$	2
$5 = \lceil \ln 5! \rceil$	8
$6 = \lfloor \ln 6! \rfloor$	8
$6 = -\left\lfloor -\sqrt{\left\lfloor \sqrt{6!} \right\rfloor} \right\rfloor$	7
$6 = \left\lceil \sqrt{\sqrt{6!}} \right\rceil$	2
$15 = \sum_{n=1}^{5} n$	6
$24 = 4! \lfloor \sqrt{2} \rfloor$	7
$24 = 4! \times \phi(2)$	6
$24 = \sqrt{(4!)^2}$	8
$25 = 5^2$	1, 3, 4, 5, 8, 9
$26 = -\lfloor -\sqrt{6!} \rfloor - \lfloor \sqrt{2} \rfloor$	7
$28 = 2 \left\lfloor \sqrt{\left\lfloor \sqrt{8!} \right\rfloor} \right\rfloor$	7
$36 = 6 \times 3!$	8, 9
$48 = 4! + (\phi(8))!$	6
$48 = 4! \lfloor \sqrt{8} \rfloor$	7
$55 = 5(-\lfloor -\sqrt{5!} \rfloor)$	7
$64 = (\sqrt{4})^6$	7, 8, 9
$70 = \lfloor \sqrt{7!} \rfloor + 0$	7, 8
$71 = \sqrt{7! + 1}$	7, 8
$72 = \lfloor \sqrt{7!} \rfloor + 2$	7, 8
$73 = \lfloor \sqrt{7!} \rfloor + 3$	7, 8
$74 = \lfloor \sqrt{7!} \rfloor + 4$	7, 8
$75 = \lfloor \sqrt{7!} \rfloor + 5$	7, 8
$76 = \lfloor \sqrt{7!} \rfloor + 6$	7, 8
$77 = \lfloor \sqrt{7!} \rfloor + 7$	7, 8
$78 = \lfloor \sqrt{7!} \rfloor + 8$	7, 8
$79 = \lfloor \sqrt{7!} \rfloor + 9$	7, 8
$81 = 1 + \left\lceil \sqrt{\sqrt{\lceil \ln(8!) \rceil !}} \right\rceil$	8
$119 = [(\sqrt{9})! - 1]! - 1$	2
$120 = (1 \div .2)! + 0$	2
$120 = (10 \div 2)!$	8
$121 = (1 \div .2)! + 1$	2
$121 = 11^2$	8, 9
$122 = (1 \div .2)! + 2$	2
$123 = (1 \div .2)! + 3$	2
$124 = (1 \div .2)! + 4$	2
$125 = (1 \div .2)! + 5$	2
$125 = 5^{2+1}$	3, 5, 7, 8, 9
$126 = 6 \times 21$	8, 9
$126 = (1 \div .2)! + 6$	2
$127 = (1 \div .2)! + 7$	2
$128 = (1 \div .2)! + 8$	2
$128 = 2^{8-1}$	4, 7, 8, 9
$129 = (1 \div .2)! + 9$	2
$135 = 3 \times 5 \div .\overline{1}$	2
$143 = 3! \times 4! - 1$	2
$144 = 3! \times 4! \times 1$	2
$144 = (1 + 4)! + 4!$	6
$144 = 4! \times (4 - 1)!$	8, 9
$145 = 3! \times 4! + 1$	2
$145 = 1! + 4! + 5!$	7, 9
$146 = \left(4! + \sqrt{.\overline{1}}\right) \times 6$	2
$147 = 7^{\sqrt{4}} \div \sqrt{.\overline{1}}$	2
$150 = 50 \div \sqrt{.\overline{1}}$	2
$152 = \sqrt[.1]{\sqrt{2}} + 5!$	2
$152 = \sum_{n=1}^{5} n! - \phi(2)$	6
$153 = 3 \times 51$	2
$154 = \sum_{n=1}^{5} n! + \phi\left(\sqrt{4}\right)$	6
$162 = \sqrt[-.2]{\sqrt{.\overline{1}}} \times .\overline{6}$	2
$168 = \sqrt{8! \times (.6 + .1)}$	2
$184 = (4! - 1) \times 8$	2
$192 = \sqrt[.1]{\sqrt{2}} \times (\sqrt{9})!$	2
$214 = 4! \div .\overline{1} - 2$	2
$216 = 6^{1+2}$	2, 3, 5, 8, 9
$225 = 5 \div (.\overline{2} - .2)$	2
$240 = (4 + 0!)! \times 2$	2
$241 = \sqrt[-.2]{\sqrt{.\overline{1}}} - \sqrt{4}$	2
$243 = 3^{\frac{2}{.4}}$	2
$242 = 3^{4+\lfloor \sqrt{1} \rfloor}$	8
$245 = \left(\sqrt{\sqrt{\sqrt{.2}}}\right)^{-(4!)} + 5!$	2
$250 = 50 \div .2 = 5 \div .02$	2
$256 = .5^{-2-6}$	2
$256 = (\lfloor \sqrt{5} \rfloor)^{6+2}$	7
$258 = .5^{-8} + 2$	2
$288 = 2^8 \div .\overline{8}$	2
$289 = (8 + 9)^2$	2, 9
$295 = 59 \div .2$	2
$315 = 35 \div .\overline{1}$	2
$324 = (4! - 3!)^2$	2, 9
$337 = 7^3 - 3!$	2

$343 = (3+4)^3$	2, 8, 9
$344 = (3!)! \times .\overline{4} + 4!$	2
$347 = 7^3 + 4$	2
$351 = (5! - 3) \div \sqrt{.\overline{1}}$	2
$354 = (5! - \sqrt{4}) \div \sqrt{.\overline{1}}$	2
$355 = (3!)! \times .5 - 5$	2
$359 = 3 \times 5! - .\overline{9}$	2
$360 = 6! \div (\lfloor\sqrt{3}\rfloor + 0!)$	7
$360 = 6! \div (3 - 0!)$	2
$384 = 8^{\sqrt{4}} \times 3!$	2
$395 = [(3!)! - 9] \times .5$	2
$432 = (3!)! \times (.4 + .2)$	2
$436 = (3!)! \times .6 + 4$	2
$456 = (5! - 6) \times 4$	2
$464 = (6! - 4!) \times \sqrt{.\overline{4}}$	2
$473 = (3!)! \times \sqrt{.\overline{4}} - 7$	2
$480 = (\sqrt{8 + 0!})! \times \sqrt{.\overline{4}}$	2
$484 = \left(\sqrt{\sqrt{4! - \sqrt{4}}}\right)^8$	2
$496 = (4! + ((\sqrt{9})!)!) \times .\overline{6}$	2
$512 = \sqrt[.\overline{1}]{\sqrt{2 \div .5}}$	2
$514 = \sqrt[-.\overline{1}]{.5} + \sqrt{4}$	2
$542 = (5! + .\overline{4}) \div .\overline{2}$	2
$584 = \sqrt[.5]{4!} + 8$	2
$595 = ((\sqrt{9})!)! - 5! - 5$	2
$599 = ((\sqrt{9})!)! - 5! - .\overline{9}$	2
$624 = 26 \times 4!$	2
$625 = 5^{6-2}$	2, 3, 7, 9
$640 = 6! - (4 + 0!)!$	9
$648 = (\sqrt{6})^8 \div \sqrt{4}$	2
$656 = 6! - .5^{-6}$	2
$660 = 6! - 60$	2
$672 = 7! \times .6 \times .\overline{2}$	2
$675 = 5! \div (.\overline{7} - .6)$	2
$688 = 86 \times 8$	2
$693 = (3!)! - \left(\sqrt{\sqrt{9}}\right)^6$	2
$696 = ((\sqrt{9})!)! - (6 \times .\overline{6})!$	2
$713 = (3!)! - 7 \times 1$	2
$715 = (7 - 1)! - 5$	2
$719 = ((\sqrt{9})!)! - 1^7$	2
$720 = (7 + 0! - 2)!$	2, 8
$720 = (7 - 0!)! \lfloor\sqrt{2}\rfloor$	7
$721 = ((\sqrt{2+7})!)! + 1$	2
$722 = ((\sqrt{2+7})!)! + 2$	2
$723 = ((\sqrt{2+7})!)! + 3$	2
$724 = ((\sqrt{2+7})!)! + 4$	2
$725 = ((\sqrt{2+7})!)! + 5$	2
$726 = ((\sqrt{2+7})!)! + 6$	2
$727 = ((\sqrt{2+7})!)! + 7$	2
$728 = ((\sqrt{2+7})!)! + 8$	2
$729 = ((\sqrt{2+7})!)! + 9$	2
$729 = 9^{\sqrt{7+2}}$	8, 9
$733 = (3!)! + 7 + 3!$	2
$734 = (3!)! + 7 \times \sqrt{4}$	2, 9
$736 = 3^6 + 7$	2
$744 = (4! + 7) \times 4!$	2
$784 = \left(\sqrt{\sqrt{4 \times 7}}\right)^8$	2
$790 = 70 + ((\sqrt{9})!)!$	2
$791 = 71 + ((\sqrt{9})!)!$	2
$792 = 72 + ((\sqrt{9})!)!$	2
$793 = 73 + ((\sqrt{9})!)!$	2
$794 = 74 + ((\sqrt{9})!)!$	2
$795 = 75 + ((\sqrt{9})!)!$	2
$796 = 76 + ((\sqrt{9})!)!$	2
$797 = 77 + ((\sqrt{9})!)!$	2
$798 = 78 + ((\sqrt{9})!)!$	2
$799 = 79 + ((\sqrt{9})!)!$	2
$809 = ((\sqrt{9})!)! \div .\overline{8} - 0!$	2
$810 = ((\sqrt{0! \div .\overline{1}})!)! \div .\overline{8}$	2
$816 = 6! \times (.8 + \sqrt{.\overline{1}})$	2
$834 = (3!)! \div .\overline{8} + 4!$	2
$864 = (\sqrt{6})^8 \times \sqrt{.\overline{4}}$	2
$895 = ((\sqrt{9})!)! \div .8 - 5$	2
$899 = ((\sqrt{9})!)! \div .8 - .\overline{9}$	2
$936 = (3!)^{\sqrt{9}} - 6!$	2
$991 = .1^{-\sqrt{9}} - 9$	2
$1024 = 2^{\lfloor\sqrt{104}\rfloor}$	7
$1024 = 2^{\sqrt{\sqrt{10^4}}}$	9
$1206 = 201 \times 6$	1
$1296 = 6^{\sqrt{9}+2-1}$	9
$1331 = 11^3 \lfloor\sqrt{3}\rfloor$	7
$1331 = 11^{\sqrt{3 \times 3}}$	9
$1440 = (4 + 1 + 0!)!\sqrt{4}$	7
$2048 = 2^{4+8-0!}$	8, 9
$2187 = (8 \div 2 - 1)^7$	4
$2401 = [(2 + 0!)! + 1]^4$	9
$2500 = 50^2 + 0$	1
$2592 = 2^5 \times 9^2$	9
$3125 = 5^{3 \cdot 1 + 2}$	7
$3125 = 5^{3 \times 2 - 1}$	3, 9
$3125 = 5^{(3+2) \times 1}$	3
$4096 = (\sqrt{4})^{6 \times (\sqrt{9} - 0!)}$	9
$5040 = (5 + 4 - 0! - 0!)!$	7, 8

$10201 = 101^2 + 0$	1
$11025 = 105^2 \times 1$	1
$12006 = 2001 \times 6$	1
$12100 = 110^2 + 0$	1
$12321 = (113 - 2)^2$	8
$14641 = 11^4 \lfloor 6 \div 4 \rfloor$	7
$14641 = 11^{4 \div .4 - 6}$	9
$15625 = 5^6 \lfloor \sqrt{5 - 2 - 1} \rfloor$	7
$15625 = 5^{6 \times 2 - 5 - 1}$	3
$15625 = 5^6 + \sqrt{5 - 1} - 2$	9
$32768 = 8^{\frac{2+6+7}{3}}$	4
$40320 = (40 - 32)!0!$	7
$40320(4 + 3 + 2 - 0!)! + 0$	8
$65536 = (5 - 6 \div 6)^{5+3}$	4
$65536 = (6 \div 3)^{5+5+6}$	8
$78125 = 5^{7 \times 2 + 1 - 8}$	4
$78125 = 5^{\frac{8+7-1}{2}}$	4
$78125 = 5^{8-1}(\lfloor \sqrt{7} \rfloor - \lfloor \sqrt{2} \rfloor$	7
$120006 = 20001 \times 6$	1
$161051 = 11^5 \cdot 16^0$	7
$250000 = 500^2 + 0 + 0$	1
$362880 = (8 + 8 + 2 - 3 - 6 - 0)!$	7
$362880 = [(8 + 8 + 6 + 2) \div 3 + 0!]!$	8
$390625 = 5^{(6 \times 2 - 9 \div 3 - 0!)}$	3
$531441 = (5 + 4)^{\frac{3+4+1}{1}}$	4
$1002001 = 1001^2 + 0 + 0$	1
$1048576 = (1 + 0!)^{8 \div 4 + 5 + 6 + 7}$	8
$1048576 = (\sqrt{4})^{6+7+8-1+0\times 5}$	8
$1048576 = (\sqrt[4]{16})^{5+7+8+0}$	8
$1200006 = 200001 \times 6$	1
$1210000 = 1100^2 + 0 + 0$	1
$1771561 = 11^6(15 - 7 - 7)$	7
$1953125 = 5^{9+5+1-1\times 2\times 3}$	3
$3628800 = (8 + 8 + 3 - 2 - 6 - 0! - 0)!$	7
$3628800 = [(8 + 8) \div 2 + 6 \div 3]! + 0 + 0$	8
$19487171 = 11^7(8 + 7 - 1 - 9 - 4)$	7
$25000000 = 5000^2 + 0 + 0 + 0$	1
$121000000 = 11000^2 + 0 + 0 + 0$	1

The editors were impressed by the ingenuity of the solutions. Hess conjectured that all numbers could be reached using combinations of the factorial, square root, floor and ceiling functions. He gives as an example 1 through 6 (in the list above). Geupel gives several infinite families: $10201, 1002001, 100020001, \ldots$*;* $2500, 250000, 25000000, \ldots$*;* $12100, 1210000, 121000000, \ldots$*; and* $1206, 12006, 120006, \ldots$.

The editors would be pleased to received more (new) representations.