

BOOK REVIEWS

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The Mathematical Mechanic: Using Physical Reasoning to Solve Problems

by Mark Levi

Princeton University Press, 2009

ISBN: 978-0-691-14020-9, Hardcover, 186 + viii pages, US\$19.95

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A mathematician and a physicist are sitting on a bench and notice a lovely lady sitting on a bench across the street.

Physicist suggests: "Let's walk over and introduce ourselves." Then he immediately starts crossing the street.

Mathematician remains seated.

Physicist turns back with a question: "Why are you not coming?"

Mathematician replies: "Well, there is no point to it. We first have to cross half the distance between us and her, then half of what is left and then half of what is left again ... We will never get there."

To this the physicist replies: "Close enough for me."

Anonymous

Often we ask ourselves whether explanations of mathematical truths that are given via physical approach are "close enough". Is physics just giving the ideas to math and math just giving the tools to physics? Is one of them either "above" the other or more needed than the other?

In the introductory chapter, Mark Levi states that: "The two subjects are so intimately intertwined that both suffer if separated." Levi goes on to say that: "In this book physics is put to work for mathematics, proving to be a very efficient servant (with apologies to physicists)".

Physical ideas implemented in very creative ways to solve math problems are the main feature of this book. The author recalls that Archimedes already used this approach in proving his famous integral calculus theorem on the volumes of a cylinder, a sphere, and a cone using an imagined balancing scale. How unfortunate it is that not many Calculus books mention this!

Following the introduction, there are 10 chapters that gradually build up the difficulty of the problems and an appendix in which the necessary physics background is presented. The reader will discover some very imaginative physical solutions to famous topics like: Pythagorean Theorem, Max and Min problems — including some beautiful calculus classics like: The Cheapest Can, The Best Spot in a Drive-in Theatre, Maneuvering a Ladder (through perpendicular hallways), Lifeguard Saving a Swimmer. Topics also go beyond usual undergraduate math classes to higher-level topics like Euler-Lagrange Equations and Gauss-Bonnet Theorem, passing through finding trigonometric derivatives and integrals, and Green's Theorem.

The author's imagination brings in "physical incarnations" of these interesting problems. Some problems invite more than one such presentation. Pythagorean Theorem offers itself in several different physical ideas — maybe because it is a topic that occurs so naturally. So we witness a proof by using a prism shaped fish tank, we "put" Pythagoras on ice and use kinetic energy as we push off the x -axis and y -axis. And then, there is a much more "explosive" proof, with a very compressed spring that is cut

and literally releases another proof of Pythagorean Theorem. My favorite is the proof done by sweeping. Levi uses electric shorting to prove inequalities between arithmetic and geometric mean. One can find centre of mass by conservation of energy, and can compute some integrals by lifting weights.

Physical illustrations are chosen from our everyday life — like spokes on a bike wheel, the path traced by a wheel of a shopping cart, even a person's nose becomes a vector. When imagination has to be stretched, illustrations are provided to describe some imaginary physical set up. It is almost tempting to go beyond just illustrations and write some java program that shows the process how a vacuum filled piston with two rings attached at its ends becomes the ladder one maneuvers through perpendicular hallways. There are a few physical incarnations though, that require almost a cartoon style imagination. The author obviously found many of these images inspiring in solving a variety of problems and the reader can feel the excitement as each topic is extended to more and more possibilities.

Have you ever wondered why negative times negative is positive? There is a thoughtful argument at the beginning of Chapter 11. This ambitious chapter prides itself in making Complex Variables Simple(r). Here Mark Levi links complex functions with idealized fluid flow in a plane and offers inspiration to those of us who need to find yet another explanation to justify to the students how imaginary numbers help us solve real world problems.

There are additional problems introduced at the end of sections, inviting the reader to use presented strategies and apply them to solve new or old favorites. Just for a teaser show that the orthogonality of eigenvectors of a symmetric matrix is a consequence of the non-existence of a perpetual motion machine. Hints, but not full solutions, are provided to many of the problems.

At the beginning of the book Levi admits that his main focus is on presenting a concept and not complete detailed proofs. Yet, only in a few places does one feel like they are really missing details. The book is not a textbook, but can serve as a supplement to instruction in providing a different approach to the material. It would, however, work only with students who have sufficient physics background. Even though the appendix claims to provide all one needs from physics, the reviewer feels that some background in calculus-based physics would definitely make the book a better read.

The book is a collection of playful ideas that present math as a subject developed in the world where physics is the axiomatic basis to sciences, to paraphrase the author. If we think of physics and mathematics as two different languages, this book confirms a well known, but underappreciated truth: it is great to be bilingual.

