

# MATHEMATICAL MAYHEM

Mathematical Mayhem began in 1988 as a **Mathematical Journal for and by High School and University Students**. It continues, with the same emphasis, as an integral part of *Crux Mathematicorum with Mathematical Mayhem*.

The interim Mayhem Editor is Shawn Godin (Cairine Wilson Secondary School, Orleans, ON). The other staff member is Monika Khbeis (Our Lady of Mt. Carmel Secondary School, Mississauga, ON).

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## Mayhem Problems

*Please send your solutions to the problems in this edition by **15 August 2011**. Solutions received after this date will only be considered if there is time before publication of the solutions.*

*Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, and 7, English will precede French, and in issues 2, 4, 6, and 8, French will precede English.*

*The editor thanks Jean-Marc Terrier of the University of Montreal for translations of the problems.*

**Note:** As *CRUX with MAYHEM* is running behind schedule, we will accept solutions past the posted due date. Solutions will be accepted until we process them for publication. Currently we are delayed by about four months. Check the CMS website, [cms.math.ca/crux](http://cms.math.ca/crux), for our status in processing problems.

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**M470.** *Proposed by the Mayhem Staff*

Vazz needs to buy desks and monitors for his new business. A desk costs **\$250** and a monitor costs **\$260**. Determine all possible ways that he could spend exactly **\$10 000** on desks and monitors.

**M471.** *Proposed by the Mayhem Staff*

Square based pyramid  $ABCDE$  has a square base  $ABCD$  with side length **10**. Its other four edges  $AE$ ,  $BE$ ,  $CE$ , and  $DE$  each have length **20**. Determine the volume of the pyramid.

**M472.** *Proposed by Neculai Stanciu, George Emil Palade Secondary School, Buzău, Romania*

Suppose that  $x$  is a real number. Without using calculus, determine the maximum possible value of  $\frac{2x^2 - 8x + 17}{x^2 - 4x + 7}$  and the minimum possible value of  $\frac{x^2 + 6x + 8}{x^2 + 6x + 10}$ .

**M473.** *Proposed by Neculai Stanciu, George Emil Palade Secondary School, Buzău, Romania*

Determine all pairs  $(a, b)$  of positive integers for which  $a^2 + b^2 - 2a + b = 5$ .

**M474.** *Proposed by Dragoljub Milošević, Gornji Milanovac, Serbia*

Let  $a, b$  and  $x$  be positive integers such that  $x^2 - bx + a - 1 = 0$ . Prove that  $a^2 - b^2$  is not a prime number.

**M475.** *Proposed by Edward T.H. Wang, Wilfrid Laurier University, Waterloo, ON*

Let  $\lfloor x \rfloor$  denote the greatest integer not exceeding  $x$ . For example,  $\lfloor 3.1 \rfloor = 3$  and  $\lfloor -1.4 \rfloor = -2$ . Let  $\{x\}$  denote the fractional part of the real number  $x$ , that is,  $\{x\} = x - \lfloor x \rfloor$ . For example,  $\{3.1\} = 0.1$  and  $\{-1.4\} = 0.6$ . Show that there exist infinitely many irrational numbers  $x$  such that  $x \cdot \{x\} = \lfloor x \rfloor$ .

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**M470.** *Proposé par l'Équipe de Mayhem*

Vazz doit acheter des pupitres et des écrans pour son nouveau commerce. Un pupitre coûte **250\$** et un écran **260\$**. Trouver de combien de manières possibles il pourrait dépenser exactement **10 000\$** en pupitres et écrans.

**M471.** *Proposé par l'Équipe de Mayhem*

Une pyramide  $ABCDE$  a une base carrée  $ABCD$  de côté **10**. Les quatre autres arêtes  $AE, BE, CE$  et  $DE$  sont toutes de longueur **20**. Trouver le volume de la pyramide.

**M472.** *Proposé par Neculai Stanciu, École secondaire George Emil Palade, Buzău, Roumanie*

Supposons que  $x$  soit un nombre réel. Sans utiliser le calcul différentiel, déterminer la valeur maximale possible de  $\frac{2x^2 - 8x + 17}{x^2 - 4x + 7}$  et la valeur minimale possible de  $\frac{x^2 + 6x + 8}{x^2 + 6x + 10}$ .

**M473.** *Proposé par Neculai Stanciu, École secondaire George Emil Palade, Buzău, Roumanie*

Déterminer toutes les paires  $(a, b)$  d'entiers positifs tels que  $a^2 + b^2 - 2a + b = 5$ .

**M474.** *Proposé par Dragoljub Milošević, Gornji Milanovac, Serbie*

Soit  $a, b$  et  $x$  trois entiers positifs tels que  $x^2 - bx + a - 1 = 0$ . Montrer que  $a^2 - b^2$  n'est pas un nombre premier.

**M475.** *Proposé par Edward T.H. Wang, Université Wilfrid Laurier, Waterloo, ON*

Notons  $\lfloor x \rfloor$  le plus grand entier n'excédant pas  $x$ . Par exemple,  $\lfloor 3,1 \rfloor = 3$  et  $\lfloor -1,4 \rfloor = -2$ . Notons  $\{x\}$  la partie fractionnaire du nombre réel  $x$ , c.-à-d,  $\{x\} = x - \lfloor x \rfloor$ . Par exemple,  $\{3,1\} = 0,1$  et  $\{-1,4\} = 0,6$ . Montrer qu'il existe une infinité de nombres irrationnels  $x$  tels que  $x \cdot \{x\} = \lfloor x \rfloor$ .

## Mayhem Solutions

**M432.** *Proposed by the Mayhem Staff.*

Determine the value of  $d$  with  $d > 0$  so that the area of the quadrilateral with vertices  $A(0, 2)$ ,  $B(4, 6)$ ,  $C(7, 5)$ , and  $D(d, 0)$  is 24.

*Solution by Edward T.H. Wang, Wilfrid Laurier University, Waterloo, ON.*

Let  $E = (0, 6)$ ,  $F = (7, 6)$ ,  $G = (7, 0)$  and  $\Omega$  denote the area function. Then

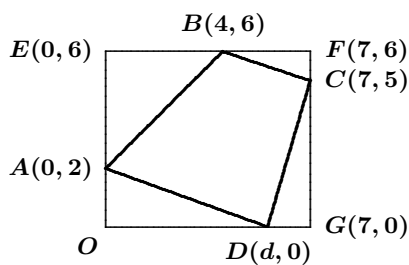
$$\Omega(AOD) = \frac{1}{2}(d \times 2) = d;$$

$$\Omega(BEA) = \frac{1}{2}(4 \times 4) = 8;$$

$$\Omega(BFC) = \frac{1}{2}(3 \times 1) = \frac{3}{2};$$

$$\text{and } \Omega(CDG) = \frac{1}{2}(7-d) \times 5 = \frac{5}{2}(7-d).$$

Since  $\Omega(OEFG) = 7 \times 6 = 42$ , we have



$$24 = \Omega(ABCD) = 42 - \left[ d + 8 + \frac{3}{2} + \frac{5}{2}(7-d) \right] = 15 + \frac{3}{2}d.$$

Solving we find  $d = 6$ .

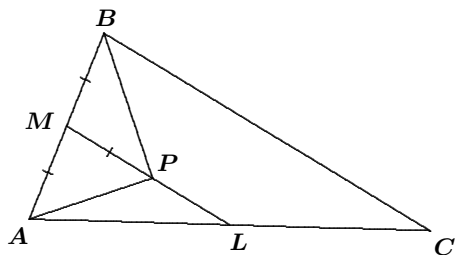
*Also solved by GEORGE APOSTOLOPOULOS, Messolonghi, Greece; NATALIA DESY, student, SMA Xaverius 1, Palembang, Indonesia; SAMUEL GÓMEZ MORENO, Universidad de Jaén, Jaén, Spain; AFIFFAH NUUR MILA HUSNIANA, student, SMPN 8, Yogyakarta, Indonesia; GEOFFREY A. KANDALL, Hamden, CT, USA; WINDA KIRANA, student, SMPN 8, Yogyakarta, Indonesia; JOSHUA LONG, Southeast Missouri State University, Cape Girardeau, MO, USA; DRAGOLJUB MILOŠEVIĆ, Gornji Milanovac, Serbia; RICARD PEIRÓ, IES "Abastos", Valencia, Spain; CAO MINH QUANG, Nguyen Binh Khiem High School, Vinh Long, Vietnam; BRUNO SALGUEIRO FANEGO, Viveiro, Spain; NECULAI STANCIU, George Emil Palade Secondary School, Buzău, Romania(2 solutions); and JOHN WYNN, student, Auburn University, Montgomery, AL, USA;*

*Two incorrect solutions were received.*

**M433.** *Proposed by Bruce Shawyer, Memorial University of Newfoundland, St. John's, NL.*

In triangle  $ABC$ ,  $AB < BC$ ,  $L$  is the midpoint of  $AC$ , and  $M$  is the midpoint of  $AB$ . Also,  $P$  is the point on  $LM$  such that  $MP = MA$ . Prove that  $\angle PBA = \angle PBC$ .

*Solution by Souparna Purohit, student, George Washington Middle School, Ridgewood, NJ, USA.*



It is well known that since  $M$  and  $L$  are the midpoints of  $AB$  and  $AC$  then  $BC \parallel ML$  so  $\angle PBC = \angle BPM$ . Also, since  $PM = AM = BM$ ,  $\triangle BMP$  is isosceles. Therefore  $\angle ABP = \angle BPM$  which, when combined with  $\angle PBC = \angle BPM$ , we conclude that  $\angle ABP = \angle PBC$ , as desired.

*Also solved by MIGUEL AMENGUAL COVAS, Cala Figuera, Mallorca, Spain (two solutions); GEORGE APOSTOLOPOULOS, Messolonghi, Greece; RICHARD I. HESS, Rancho Palos Verdes, CA, USA; GEOFFREY A. KANDALL, Hamden, CT, USA; WINDA KIRANA, student, SMPN 8, Yogyakarta, Indonesia; DRAGOLJUB MILOŠEVIĆ, Gornji Milanovac, Serbia; RICARD PEIRÓ, IES "Abastos", Valencia, Spain; BRUNO SALGUEIRO FANEGO, Viveiro, Spain; HUGO LUYO SÁNCHEZ, Pontificia Universidad Católica del Perú, Lima, Peru; NECULAI STANCIU, George Emil Palade Secondary School, Buzău, Romania; EDWARD T.H. WANG, Wilfrid Laurier University, Waterloo, ON; and the proposer.*

*One incorrect solution was received. Several readers pointed out that  $\triangle APB$  is right angled with the right angle at  $P$ .*

**M434.** *Proposed by Heisu Nicolae, Pîrjol Secondary School, Bacău, Romania.*

Determine all eight-digit positive integers  $abcdefgh$  which satisfy the relations  $a^3 - b^2 = 2$ ,  $c^3 - d^2 = 4$ ,  $2^e - f^2 = 7$ , and  $g^3 - h^2 = -1$ .

*Solution by Arkady Alt, San Jose, CA, USA.*

Since  $2 \leq a^3 \leq 9^2 + 2 = 83 \Leftrightarrow 2 \leq a \leq 4$  and  $a^3 - 2$  for such  $a$  can only be square for  $a = 3$ , then  $a = 3, b = 5$ .

Since  $4 \leq c^3 \leq 9^2 + 4 = 85 \Leftrightarrow 2 \leq c \leq 4$  and  $c^3 - 4$  for such  $c$  can only be square for  $c = 2$  then  $c = 2, d = 2$ .

Since  $7 \leq 2^e \leq 9^2 + 7 = 88 \Leftrightarrow 3 \leq e \leq 6$  and  $2^e - 7$  for such  $e$  can only be square for  $e = 3, e = 4$  and  $e = 5$  then  $(e, f) = (3, 1), (4, 3), (5, 5)$ .

Since  $0 \leq g^3 \leq 9^2 - 1 = 80 \Leftrightarrow 0 \leq g \leq 4$  and  $g^3 + 1$  for such  $g$  can only be square for  $g = 0$  and  $g = 2$  then  $(g, h) = (0, 1), (2, 3)$ .

Thus  $abcdefgh = 35\ 223\ 101, 35\ 224\ 301, 35\ 225\ 501, 35\ 223\ 123, 35\ 224\ 323, 35\ 225\ 523$ .

*Also solved by RICHARD I. HESS, Rancho Palos Verdes, CA, USA; RICARD PEIRÓ, IES "Abastos", Valencia, Spain; CAO MINH QUANG, Nguyen Binh Khiem High School, Vinh Long, Vietnam; BRUNO SALGUEIRO FANEGO, Viveiro, Spain; EDWARD T.H. WANG, Wilfrid Laurier University, Waterloo, ON; and the proposer. Seven incomplete solutions were submitted. Most of the incomplete solutions missed the case where  $g = 0$ .*

**M435.** Proposed by Mihály Bencze, Brasov, Romania.

Prove that

$$\sum_{k=1}^n \sqrt{1 + \frac{1}{k^2} + \frac{1}{(k+1)^2}} = \frac{n(n+2)}{n+1}.$$

*Solution by Cao Minh Quang, Nguyen Binh Khiem High School, Vinh Long, Vietnam.*

For any  $k > 0$  we have

$$\begin{aligned} \left(1 + \frac{1}{k} - \frac{1}{k+1}\right)^2 &= 1 + \frac{1}{k^2} + \frac{1}{(k+1)^2} + \left(\frac{2}{k} - \frac{2}{k+1}\right) - \frac{2}{k(k+1)} \\ &= 1 + \frac{1}{k^2} + \frac{1}{(k+1)^2} + \frac{2}{k(k+1)} - \frac{2}{k(k+1)} \\ &= 1 + \frac{1}{k^2} + \frac{1}{(k+1)^2}. \end{aligned}$$

Hence  $\sqrt{1 + \frac{1}{k^2} + \frac{1}{(k+1)^2}} = 1 + \frac{1}{k} - \frac{1}{k+1}$ . Therefore if we let  $S = \sum_{k=1}^n \sqrt{1 + \frac{1}{k^2} + \frac{1}{(k+1)^2}}$  then

$$\begin{aligned} S &= \sum_{k=1}^n \left(1 + \frac{1}{k} - \frac{1}{k+1}\right) \\ &= \left(1 + \frac{1}{1} - \frac{1}{2}\right) + \left(1 + \frac{1}{2} - \frac{1}{3}\right) + \cdots + \left(1 + \frac{1}{n} - \frac{1}{n+1}\right) \\ &= n + 1 - \frac{1}{n+1} = \frac{n(n+2)}{n+1}, \end{aligned}$$

and we are done!

*Also solved by* ARKADY ALT, San Jose, CA, USA; MIGUEL AMENGUAL COVAS, Cala Figuera, Mallorca, Spain; GEORGE APOSTOLOPOULOS, Messolonghi, Greece; SAMUEL GÓMEZ MORENO, Universidad de Jaén, Jaén, Spain; G.C. GREUBEL, Newport News, VA, USA; RICHARD I. HESS, Rancho Palos Verdes, CA, USA; GEOFFREY A. KANDALL, Hamden, CT, USA; WINDA KIRANA, student, SMPN 8, Yogyakarta, Indonesia; HUGO LUYO SÁNCHEZ, Pontificia Universidad Católica del Perú, Lima, Peru; DAVID E. MANES, SUNY at Oneonta, Oneonta, NY, USA; PEDRO HENRIQUE O. PANTOJA, student, UFRN, Brazil; RICARD PEIRÓ, IES "Abastos", Valencia, Spain; PAOLO PERFETTI, Dipartimento di Matematica, Università degli studi di Tor Vergata Roma, Rome, Italy; DRAGOLJUB MILOŠEVIĆ, Gornji Milanovac, Serbia; NATALIA DESY, student, SMA Xaverius 1, Palembang, Indonesia; BRUNO SALGUEIRO FANEGO, Viveiro, Spain; NECULAI STANCIU, George Emil Palade Secondary School, Buzău, Romania; EDWARD T.H. WANG, Wilfrid Laurier University, Waterloo, ON; and the proposer.

**M436.** Proposed by Neculai Stanciu, George Emil Palade Secondary School, Buzău, Romania.

Determine the smallest possible value of  $x + y$ , if  $x$  and  $y$  are positive integers with  $\frac{2008}{2009} < \frac{x}{y} < \frac{2009}{2010}$ .

*Solution by Richard I. Hess, Rancho Palos Verdes, CA, USA.*

Let  $y = x + d$  then  $1 - \frac{1}{2009} < \frac{y-d}{y} < 1 - \frac{1}{2010}$  so  $\frac{1}{2009} > \frac{d}{y} > \frac{1}{2010}$ . If  $d = 1$  there is no solution. If  $d = 2$ ,  $y = 4019$  is a solution so  $x = 4017$  and  $x + y = 8036$ . If  $d > 2$  then  $x, y > 6000$  thus  $x + y > 12000$ . Therefore the minimum value of  $x + y$  is **8036**.

*Also solved by ARKADY ALT, San Jose, CA, USA; SAMUEL GÓMEZ MORENO, Universidad de Jaén, Jaén, Spain; CAO MINH QUANG, Nguyen Binh Khiem High School, Vinh Long, Vietnam; DAVID E. MANES, SUNY at Oneonta, Oneonta, NY, USA; EDWARD T.H. WANG, Wilfrid Laurier University, Waterloo, ON; and the proposer. Five incorrect solutions were submitted.*

*Alt, Manes and Wang proved in general that if  $\frac{n}{n+1} < \frac{x}{y} < \frac{n+1}{n+2}$  then the solution with the smallest sum corresponds to  $x = 2n + 1$ ,  $y = 2n + 3$  and thus  $x + y = 4n + 4$ . In general for any two fractions of non-negative integers, in lowest terms,  $\frac{a}{b} < \frac{c}{d}$  the value  $\frac{a+c}{b+d}$  is called the mediant and it satisfies  $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$ . If we also have  $bc - ad = 1$  then the mediant is the fraction with the lowest denominator in the interval  $(\frac{a}{b}, \frac{c}{d})$ .*

**M437.** Proposed by Samuel Gómez Moreno, Universidad de Jaén, Jaén, Spain.

Let  $\lfloor x \rfloor$  denote the greatest integer not exceeding  $x$ . For example,  $\lfloor 3.1 \rfloor = 3$  and  $\lfloor -1.4 \rfloor = -2$ . Let  $\{x\}$  denote the fractional part of the real number  $x$ , that is,  $\{x\} = x - \lfloor x \rfloor$ . For example,  $\{3.1\} = 0.1$  and  $\{-1.4\} = 0.6$ . Determine all rational numbers  $x$  such that  $x \cdot \{x\} = \lfloor x \rfloor$ .

*Solution by David E. Manes, SUNY at Oneonta, Oneonta, NY, USA.*

The only rational number  $x$  such that  $x \cdot \{x\} = \lfloor x \rfloor$  is  $x = 0$ .

If  $n$  is an integer, then  $n \cdot \{n\} = 0 = \lfloor n \rfloor = n$  has the only solution  $n = 0$ . Therefore,  $0$  is the only integer solution to the equation.

Assume  $x$  is a rational number different from an integer such that  $x \cdot \{x\} = \lfloor x \rfloor = x - \{x\}$ , then  $\{x\} = \frac{x}{x+1}$ . Therefore,  $x \left( \frac{x}{x+1} \right) = \lfloor x \rfloor$  implies  $x^2 = \lfloor x \rfloor(x + 1)$ . Assume  $x = \frac{m}{n}$  where  $m$  and  $n$  are relatively prime integers and  $n > 1$ . Then

$$\frac{m^2}{n^2} = \lfloor x \rfloor \left( \frac{m}{n} + 1 \right) = \lfloor x \rfloor \left( \frac{m+n}{n} \right).$$

As a result,  $m^2 = \lfloor x \rfloor(m+n) \cdot n$  so that  $n$  is a divisor of  $m^2$ , a contradiction since  $m$  and  $n$  are relatively prime and  $n > 1$ .

*Also solved by ARKADY ALT, San Jose, CA, USA; RICHARD I. HESS, Rancho Palos Verdes, CA, USA; RICARD PEIRÓ, IES "Abastos", Valencia, Spain; EDWARD T.H. WANG, Wilfrid Laurier University, Waterloo, ON; and the proposer. Three incorrect solutions were submitted.*

# Problem of the Month

Ian VanderBurgh

Problems involving averages and their properties appear frequently on contests (*Look at the solution to question 1 of Skoliad on page 5 – Ed.*). This month and next, we will look at a few of these problems, at least one of which uses averages in a very subtle way.

**Problem 1** (2008 Small c Contest) The average of three numbers is **13**. Two numbers are added to this list so that the average of all five numbers is **17**. What is the average of the two new numbers?

(A) 21 (B) 25 (C) 23 (D) 30 (E) 15

One of the things about average problems that I like is that there are really only about  $1\frac{1}{2}$  things that you need to know about averages in order to be able to do almost all of such problems. (That's not to say that there isn't a plethora of tricks of the trade that can be useful...)

The first of these  $1\frac{1}{2}$  important things is how to calculate an average: add up the given numbers, count the given numbers, and divide the sum by the count to get the average. The extra  $\frac{1}{2}$  thing to remember is that the sum of the numbers equals the count times the average. Expressing these facts algebraically, we see that if there are  $n$  numbers whose sum is  $S$ , then the average,  $a$ , satisfies the equation  $a = \frac{S}{n}$ . Rearranging this gives  $S = na$ . (I concede that occasionally we might use the fact that  $n = \frac{S}{a}$  as well.)

Let's solve Problem 1 using these properties and then look at our answer to see what we can observe.

**Solution to Problem 1.** Since the average of the original three numbers is **13**, then their sum is  $3 \times 13 = 39$ . Since the average of all five numbers is **17**, then the sum of the five numbers is  $5 \times 17 = 85$ .

The sum of the additional two numbers equals the sum of all five numbers minus the sum of the original three numbers, or  $85 - 39 = 46$ . Therefore, the average of these two numbers is  $\frac{46}{2} = 23$ . ■

This problem is particularly nice, in my opinion, because it doesn't require us to use any algebra. Let's look at the data that we have:

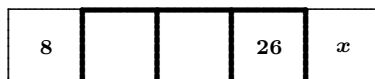
- the average of the first **3** numbers is **13**
- the average of all **5** numbers is **17**
- the average of the last **2** numbers is **23**

Do you notice anything about the position of the overall average relative to the averages of the first and last numbers? You might have noticed that the overall average splits these averages in the ratio  $4 : 6$  which equals  $2 : 3$ , which happens to be the ratio of the count of numbers in each partial average (arranged in reverse from what you might quickly guess).

If this rule works in general, then if we had **5** numbers with average **22** and **3** numbers with average **46**, the average of all **8** numbers should split **22** and **46** in the ratio  $3 : 5$ . In other words, the average is  $\frac{3}{3+5} = \frac{3}{8}$  of the way from **22** to **46**, and so equals  $22 + \frac{3}{8} \times (46 - 22) = 31$ . Try solving this problem using the method that we used above to confirm the answer.

Putting this in a more general way, if  $m$  numbers have an average of  $a$  and  $n$  numbers have an average of  $b$  with  $a < b$ , then the average of the  $m + n$  numbers splits  $a$  and  $b$  in the ratio  $n : m$  (not  $m : n$ ). Can you prove this? We'll look at another problem next month where this approach is really useful.

**Problem 2** (2010 Pascal Contest) In the diagram, each of the five boxes is to contain a number. Each number in a bold outlined box must be the average of the number in the box to the left of it and the number in the box to the right of it. What is the value of  $x$ ?



- (A) 28 (B) 30 (C) 31 (D) 32 (E) 34

Special cases often produce interesting facts. For example, if two numbers  $x$  and  $y$  have an average of  $a$ , then  $\frac{x+y}{2} = a$  or  $x+y = 2a$ . Try to use this to solve the following problem algebraically.

**Solution to Problem 2.** We label the numbers in the empty boxes as  $y$  and  $z$ , so the numbers in the boxes are thus **8,  $y$ ,  $z$ , 26,  $x$** .

Since the average of  $z$  and  $x$  is 26, then  $x+z = 2(26) = 52$  or  $z = 52-x$ . We rewrite the list as **8,  $y$ ,  $52-x$ , 26,  $x$** .

Since the average of **26** and  $y$  is  $52-x$ , then  $26+y = 2(52-x)$  or  $y = 104 - 26 - 2x = 78 - 2x$ . We rewrite the list as **8,  $78-2x$ ,  $52-x$ , 26,  $x$** .

Since the average of **8** and  $52-x$  is  $78-2x$ , then  $8+(52-x) = 2(78-2x)$  or  $60-x = 156-4x$  and so  $3x = 96$  or  $x = 32$ . ■

Especially while writing a contest, it's very tempting to take the answer that we get and not think about it at all. But let's actually take this a moment to use this answer and go back to the list in terms of  $x$  (written as **8,  $78-2x$ ,  $52-x$ , 26,  $x$** ) and substitute to get the list **8, 14, 20, 26, 32**.



Do you recognize what kind of sequence this list forms? This is an arithmetic sequence. (Look up this term if you've never seen it before.) Do you think that this is a coincidence? (Hint: The answer to this question is almost always no.)

Let's think about this by going back to the list  $\mathbf{8, y, z, 26, x}$ . Let's avoid using algebra, but we'll keep these labels to make things a little clearer. We are told that  $\mathbf{y}$  is the average of  $\mathbf{8}$  and  $\mathbf{z}$ . The important fact to recognize here is that  $\mathbf{y}$  is halfway between  $\mathbf{8}$  and  $\mathbf{z}$ . In other words, the difference  $\mathbf{y - 8}$  equals  $\mathbf{z - y}$ . Similarly,  $\mathbf{z}$  is the average of  $\mathbf{26}$  and  $\mathbf{y}$ , so  $\mathbf{26 - z}$  equals  $\mathbf{z - y}$ . But there is a common difference in these two sentences! (And it's no coincidence that I used the phrase common difference...)

Since there is this common difference, then all three differences must be equal. Since  $\mathbf{26 - 8 = 18}$ , then each of these differences equals  $\mathbf{18 \div 3 = 6}$ , and so the numbers in the sequence are  $\mathbf{8, 14, 20, 26, x}$ . Can you extend this argument another step to explain why  $\mathbf{x = 32}$ ?

So what is the connection between averages and arithmetic sequences? An arithmetic sequence is a sequence with the property that each term after the first is the average of the term before and the term after. This is pretty neat, if you've never seen it before. One last thing to think about – the average is sometimes called the *arithmetic* mean. Coincidence?




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Adams, Douglas (1952 - 2001) The first nonabsolute number is the number of people for whom the table is reserved. This will vary during the course of the first three telephone calls to the restaurant, and then bear no apparent relation to the number of people who actually turn up, or to the number of people who subsequently join them after the show/match/party/gig, or to the number of people who leave when they see who else has turned up. The second nonabsolute number is the given time of arrival, which is now known to be one of the most bizarre of mathematical concepts, a reciprivertexcluson, a number whose existence can only be defined as being anything other than itself. In other words, the given time of arrival is the one moment of time at which it is impossible that any member of the party will arrive. Reciprivertexclusons now play a vital part in many branches of math, including statistics and accountancy and also form the basic equations used to engineer the Somebody Else's Problem field. The third and most mysterious piece of nonabsoluteness of all lies in the relationship between the number of items on the bill, the cost of each item, the number of people at the table and what they are each prepared to pay for. (The number of people who have actually brought any money is only a subphenomenon of this field.) "Life, the Universe and Everything." New York: Harmony Books, 1982.