MATHEMATICAL MAYHEM

Mathematical Mayhem began in 1988 as **a Mathematical Journal for and by High School and University Students**. It continues, with the same emphasis, as an integral part of ***Crux Mathematicorum with Mathematical Mayhem***.

The interim Mayhem Editor is Shawn Godin (Cairine Wilson Secondary School, Orleans, ON). The other staff member is Monika Khbeis (Our Lady of Mt. Carmel Secondary School, Mississauga, ON).

Mayhem Problems

*Please send your solutions to the problems in this edition by **15 August 2011**. Solutions received after this date will only be considered if there is time before publication of the solutions.*

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, and 7, English will precede French, and in issues 2, 4, 6, and 8, French will precede English.

The editor thanks Jean-Marc Terrier of the University of Montreal for translations of the problems.

Note: As ***CRUX with MAYHEM*** is running behind schedule, we will accept solutions past the posted due date. Solutions will be accepted until we process them for publication. Currently we are delayed by about four months. Check the CMS website, **cms.math.ca/crux**, for our status in processing problems.

M470. *Proposed by the Mayhem Staff*

Vazz needs to buy desks and monitors for his new business. A desk costs $\$250$ and a monitor costs $\$260$. Determine all possible ways that he could spend exactly $\$10\,000$ on desks and monitors.

M471. *Proposed by the Mayhem Staff*

Square based pyramid $ABCDE$ has a square base $ABCD$ with side length 10. Its other four edges AE, BE, CE, and DE each have length 20. Determine the volume of the pyramid.

M472. *Proposed by Neculai Stanciu, George Emil Palade Secondary School, Buzău, Romania*

Suppose that x is a real number. Without using calculus, determine the maximum possible value of $\dfrac{2x^2-8x+17}{x^2-4x+7}$ and the minimum possible value of $\dfrac{x^2+6x+8}{x^2+6x+10}$.

M473. *Proposed by Neculai Stanciu, George Emil Palade Secondary School, Buzău, Romania*

Determine all pairs (a, b) of positive integers for which $a^2+b^2-2a+b = 5$.

M474. *Proposed by Dragoljub Milošević, Gornji Milanovac, Serbia*

Let a, b and x be positive integers such that $x^2 - bx + a - 1 = 0$. Prove that $a^2 - b^2$ is not a prime number.

M475. *Proposed by Edward T.H. Wang, Wilfrid Laurier University, Waterloo, ON*

Let $\lfloor x \rfloor$ denote the greatest integer not exceeding x. For example, $\lfloor 3.1 \rfloor = 3$ and $\lfloor -1.4 \rfloor = -2$. Let $\{x\}$ denote the fractional part of the real number x, that is, $\{x\} = x - \lfloor x \rfloor$. For example, $\{3.1\} = 0.1$ and $\{-1.4\} = 0.6$. Show that there exist infinitely many irrational numbers x such that $x \cdot \{x\} = \lfloor x \rfloor$.

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M470. *Proposé par l'Équipe de Mayhem*

Vazz doit acheter des pupitres et des écrans pour son nouveau commerce. Un pupitre coûte **250$** et un écran **260$**. Trouver de combien de manières possibles il pourrait dépenser exactement **10 000$** en pupitres et écrans.

M471. *Proposé par l'Équipe de Mayhem*

Une pyramide $ABCDE$ a une base carrée $ABCD$ de côté 10. Les quatre autres arêtes AE, BE, CE et DE sont toutes de longueur 20. Trouver le volume de la pyramide.

M472. *Proposé par Neculai Stanciu, École secondaire George Emil Palade, Buzău, Roumanie*

Supposons que x soit un nombre réel. Sans utiliser le calcul différentiel, déterminer la valeur maximale possible de $\dfrac{2x^2 - 8x + 17}{x^2 - 4x + 7}$ et la valeur minimale possible de $\dfrac{x^2 + 6x + 8}{x^2 + 6x + 10}$.

M473. *Proposé par Neculai Stanciu, École secondaire George Emil Palade, Buzău, Roumanie*

Déterminer toutes les paires (a, b) d'entiers positifs tels que $a^2 + b^2 - 2a + b = 5$.

M474. *Proposé par Dragoljub Milošević, Gornji Milanovac, Serbie*

Soit a, b et x trois entiers positis tels que $x^2 - bx + a - 1 = 0$. Montrer que $a^2 - b^2$ n'est pas un nombre premier.

M475. *Proposé par Edward T.H. Wang, Université Wilfrid Laurier, Waterloo, ON*

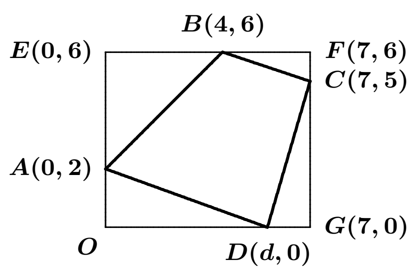
Notons $\lfloor x \rfloor$ le plus grand entier n'excédant pas x. Par exemple, $\lfloor 3{,}1 \rfloor = 3$ et $\lfloor -1{,}4 \rfloor = -2$. Notons $\{x\}$ la partie fractionnaire du nombre réel x, c.-à-d, $\{x\} = x - \lfloor x \rfloor$. Par exemple, $\{3{,}1\} = 0{,}1$ et $\{-1{,}4\} = 0{,}6$. Montrer qu'il existe une infinité de nombres irrationnels x tels que $x \cdot \{x\} = \lfloor x \rfloor$.

Mayhem Solutions

M432. *Proposed by the Mayhem Staff.*

Determine the value of d with $d > 0$ so that the area of the quadrilateral with vertices $A(0,2)$, $B(4,6)$, $C(7,5)$, and $D(d,0)$ is 24.

Solution by Edward T.H. Wang, Wilfrid Laurier University, Waterloo, ON.



Let $E = (0,6), F = (7,6), G = (7,0)$ and Ω denote the area function. Then $\Omega(AOD) = \frac{1}{2}(d \times 2) = d;$ $\Omega(BEA) = \frac{1}{2}(4 \times 4) = 8;$ $\Omega(BFC) = \frac{1}{2}(3 \times 1) = \frac{3}{2};$ and $\Omega(CDG) = \frac{1}{2}(7-d)\times 5 = \frac{5}{2}(7-d)$. Since $\Omega(OEFG) = 7 \times 6 = 42$, we have

$$24 = \Omega(ABCD) = 42 - \left[d + 8 + \frac{3}{2} + \frac{5}{2}(7-d)\right] = 15 + \frac{3}{2}d.$$

Solving we find $d = 6$.

Also solved by GEORGE APOSTOLOPOULOS, Messolonghi, Greece; NATALIA DESY, student, SMA Xaverius 1, Palembang, Indonesia; SAMUEL GÓMEZ MORENO, Universidad de Jaén, Jaén, Spain; AFIFFAH NUUR MILA HUSNIANA, student, SMPN 8, Yogyakarta, Indonesia; GEOFFREY A. KANDALL, Hamden, CT, USA; WINDA KIRANA, student, SMPN 8, Yogyakarta, Indonesia; JOSHUA LONG, Southeast Missouri State University, Cape Girardeau, MO, USA; DRAGOLJUB MILOŠEVIĆ, Gornji Milanovac, Serbia; RICARD PEIRÓ, IES "Abastos", Valencia, Spain; CAO MINH QUANG, Nguyen Binh Khiem High School, Vinh Long, Vietnam; BRUNO SALGUEIRO FANEGO, Viveiro, Spain; NECULAI STANCIU, George Emil Palade Secondary School, Buzău, Romania(2 solutions); and JOHN WYNN, student, Auburn University, Montgomery, AL, USA;

Two incorrect solutions were received.

M433. *Proposed by Bruce Shawyer, Memorial University of Newfoundland, St. John's, NL.*

In triangle ABC, $AB < BC$, L is the midpoint of AC, and M is the midpoint of AB. Also, P is the point on LM such that $MP = MA$. Prove that $\angle PBA = \angle PBC$.