MATHEMATICAL MAYHEM

Mathematical Mayhem began in 1988 as **a Mathematical Journal for and by High School and University Students**. It continues, with the same emphasis, as an integral part of ***Crux Mathematicorum with Mathematical Mayhem***.

The Mayhem Editor is Ian VanderBurgh (University of Waterloo). The other staff members are Monika Khbeis (Our Lady of Mt. Carmel Secondary School, Mississauga, ON) and Eric Robert (Leo Hayes High School, Fredericton, NB).

Mayhem Problems

*Please send your solutions to the problems in this edition by **1 May 2010**. Solutions received after this date will only be considered if there is time before publication of the solutions. The Mayhem Staff ask that each solution be submitted on a separate page and that the solver's name and contact information be included with each solution.*

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, and 7, English will precede French, and in issues 2, 4, 6, and 8, French will precede English.

The editor thanks Jean-Marc Terrier of the University of Montreal for translations of the problems.

M420. *Proposed by the Mayhem Staff.*

Riley is a poor starving university student, but is mathematically astute. He notices that five suppers in residence cost the same as seven lunches. After one week of skipping supper most nights, he notices that five lunches and one supper cost $\$48$ in total. How much do 16 suppers cost?

M421. *Proposed by Neculai Stanciu, George Emil Palade Secondary School, Buzău, Romania.*

Let $\lfloor x \rfloor$ be the greatest integer less than or equal to the real number x. Determine all real numbers x such that

$$\left\lfloor \frac{1}{x} \right\rfloor + \left\lfloor \frac{3}{x} \right\rfloor \;=\; 4\,.$$

M422. *Proposed by Adnan Arapovic, student, University of Waterloo, Waterloo, ON.*

Prove that

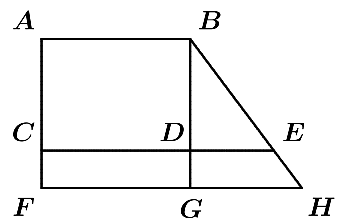
$$\sum_{k=1}^{n} \frac{k(k+1)}{2} \;=\; \frac{n(n+1)(n+2)}{6}\,.$$

M423. *Proposed by John Grant McLoughlin, University of New Brunswick, Fredericton, NB.*

The tens digit of a perfect square S is three greater than the ones digit of S. Determine all possible remainders when S is divided by 100.

M424. *Proposed by Margo Kondratieva, Memorial University of Newfoundland, St. John's, NL.*

In the diagram, line segments AB, CDE, and FGH are parallel. Also, line segments ACF and BDG are perpendicular to AB. Suppose that the area of rectangle $ABDC$ is x, the area of rectangle $CDGF$ is y, and the area of $\triangle BDE$ is z. Determine the area of $DEHG$ in terms of x, y, and z.



M425. *Proposed by Titu Zvonaru, Comănești, Romania.*

In $\triangle ABC$, $\angle BAC = 90^\circ$ and I is the incentre. The interior bisector of angle C meets AB at D. The line segment through D perpendicular to BI intersects BC at E. The line segment through D parallel to BI meets AC at F. Prove that E, I, and F are collinear.

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M420. *Proposé par l'Équipe de Mayhem.*

Richard est un étudiant pauvre et affamé, mais mathématiquement doué. Il a remarqué qu'à la résidence, cinq soupers coûtent le même prix que sept lunchs. Après avoir sauté les soupers presque tous les soirs pendant une semaine, il constate que cinq lunchs et un souper coûtent 48 au total. Combien coûtent 16 soupers ?

M421. *Proposé par Neculai Stanciu, École secondaire George Emil Palade, Buzău, Roumanie.*

Soit $\lfloor x \rfloor$ le plus grand entier plus petit ou égal au nombre réel x. Trouver tous les nombres réels tels que

$$\left\lfloor \frac{1}{x} \right\rfloor + \left\lfloor \frac{3}{x} \right\rfloor = 4\,.$$

M422. *Proposé par Adnan Arapovic, étudiant, Université de Waterloo, Waterloo, ON.*

Montrer que

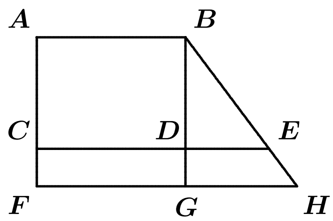
$$\sum_{k=1}^{n} \frac{k(k+1)}{2} = \frac{n(n+1)(n+2)}{6}\,.$$

M423. *Proposé par John Grant McLoughlin, Université du Nouveau-Brunswick, Fredericton, NB.*

La différence entre le chiffre des dizaines et celui des unités d'un carré parfait S est de trois. Trouver tous les restes de la division de S par 100.

M424. *Proposé par Margo Kondratieva, Université Memorial de Terre-Neuve, St. John's, NL.*

Dans la figure ci-contre, les segments de droite AB, CDE, et FGH sont parallèles. De plus, les segments ACF et BDG sont perpendiculaires à AB. Supposons que les aires respectives des rectangles $ABDC$, $CDGF$, et $\triangle BDE$ sont x, y et z. Trouver l'aire de $DEHG$ en fonction de x, y et z.



M425. *Proposé par Titu Zvonaru, Comăneşti, Roumanie.*

Dans le triangle ABC, $\angle BAC = 90°$ et soit I le centre du cercle inscrit. La bissectrice intérieure de l'angle C coupe AB en D. La droite passant par D et perpendiculaire à BI coupe BC en E. La droite passant par D et parallèle à BI coupe AC en F. Montrer que E, I et F sont colinéaires.

Mayhem Solutions

Last year we received some late solutions that did not appear in the December issue. We therefore acknowledge a correct solution to M383 by Mridul Singh, student, Kendriya Vidyalaya School, Shillong, India, and correct solutions to problems M383, M384, and M386 by Hugo Luyo Sánchez, Pontificia Universidad Católica del Peru, Lima, Peru.

M388. *Proposed by Kyle Sampson, St. John's, NL.*

A sequence is generated by listing (from smallest to largest) for each positive integer n the multiples of n up to and including n^2. Thus, the sequence begins $1, 2, 4, 3, 6, 9, 4, 8, 12, 16, 5, 10, 15, 20, 25, 6, 12, \dots$. Determine the 2009^{th} term in the sequence.

Solution by Kristóf Huszár, Valéria Koch Grammar School, Pécs, Hungary.

First, we notice that there are k positive integral multiples of k less than or equal to k^2. If we group the terms of the sequence as the multiples of 1, then the multiples of 2, then the multiples 3, and so on, we notice that the groups have 1 term, then 2 terms, then 3 terms, and so on.