MATHEMATICAL MAYHEM

Mathematical Mayhem began in 1988 as **a Mathematical Journal for and by High School and University Students**. It continues, with the same emphasis, as an integral part of ***Crux Mathematicorum with Mathematical Mayhem***.

The Mayhem Editor is Ian VanderBurgh (University of Waterloo). The other staff members are Monika Khbeis (Ascension of Our Lord Secondary School, Mississauga) and Eric Robert (Leo Hayes High School, Fredericton).

Mayhem Problems

Please send your solutions to the problems in this edition by **1 May 2009**. *Solutions received after this date will only be considered if there is time before publication of the solutions.*

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, and 7, English will precede French, and in issues 2, 4, 6, and 8, French will precede English.

The editor thanks Jean-Marc Terrier of the University of Montreal for translations of the problems.

M376. *Proposed by the Mayhem Staff.*

Determine the value of x if $\left(10^{2009}+25\right)^2-\left(10^{2009}-25\right)^2=10^x$.

M377. *Proposed by the Mayhem Staff.*

An arithmetic sequence consists of 9 positive integers. The sum of the terms in the sequence is greater than 200 and less than 220. If the second term in the sequence is 12, determine the sequence.

M378. *Proposed by the Mayhem Staff.*

Points C and D are chosen on the semi-circle with diameter AB so that C is closer to A. Segments CB and DA intersect at P; segments AC and BD extended intersect at Q. Prove that QP extended is perpendicular to AB.

M379. *Proposed by John Grant McLoughlin, University of New Brunswick, Fredericton, NB.*

The integers $27+C$, $555+C$, and $1371+C$ are all perfect squares, the square roots of which form an arithmetic sequence. Determine all possible values of C.

M380. *Proposed by Bruce Shawyer, Memorial University of Newfoundland, St. John's, NL.*

Triangle ABC is right-angled at C and has $BC = a$ and $CA = b$, with $a \geq b$. Squares $ABDE$, $BCFG$, and $CAHI$ are drawn externally to triangle ABC. The lines through FI and EH intersect at P, the lines through FI and DG intersect at Q, and the lines through DG and EH intersect at R. If triangle PQR is right-angled, determine the value of $\frac{b}{a}$.

M381. *Proposed by Mihály Bencze, Brasov, Romania.*

Determine all solutions to the equation

$$\frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-6} + \frac{1}{x-7} = x^2 - 4x - 4\,.$$

. . . .

M376. *Proposé par l'Équipe de Mayhem.*

Déterminer la valeur de x si $\left(10^{2009} + 25\right)^2 - \left(10^{2009} - 25\right)^2 = 10^x$.

M377. *Proposé par l'Équipe de Mayhem.*

Déterminer la suite arithmétique formée de 9 entiers positifs dont la somme se situe entre 200 et 220 et dont le second terme vaut 12.

M378. *Proposé par l'Équipe de Mayhem.*

A partir du point A, on choisit deux points C et D sur un demi-cercle de diamètre AB. Soit P l'intersection des droites CB et DA, et Q celle des droites AC et BD. Montrer que la droite PQ est perpendiculaire à AB.

M379. *Proposé par John Grant McLoughlin, Université du Nouveau-Brunswick, Fredericton, NB.*

Les entiers $27 + C$, $555 + C$ et $1371 + C$ sont tous des carrés parfaits dont les racines carrées forment une suite arithmétique. Trouver toutes les valeurs possibles de C.

M380. *Proposé par Bruce Shawyer, Université Memorial de Terre-Neuve, St. John's, NL.*

Dans un triangle ABC d'angle droit en C, soit $BC = a$ et $CA = b$, avec $a \geq b$. Extérieurement au triangle ABC, on construit les carrés $ABDE$, $BCFG$ et $CAHI$. Soit respectivement P, Q et R les intersections des droites FI et EH, FI et DG, DG et EH. Déterminer la valeur de $\frac{b}{a}$ pour que PQR soit un triangle rectangle.

M381. *Proposé par Mihály Bencze, Brasov, Roumanie.*

Déterminer toutes les solutions de l'équation

$$\frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-6} + \frac{1}{x-7} = x^2 - 4x - 4\,.$$

Mayhem Solutions

M338. *Proposed by the Mayhem Staff.*

Two students miscopy the quadratic equation $x^2 + bx + c = 0$ that their teacher writes on the board. Jim copies b correctly but miscopies c; his equation has roots 5 and 4. Vazz copies c correctly, but miscopies b; his equation has roots 2 and 4. What are the roots of the original equation?

Solution by Taylor Thetford, student, Lakeview High School, San Angelo, TX, USA.

The roots of the quadratic equation that Jim writes down are 5 and 4. His quadratic equation is thus $(x-5)(x-4) = x^2 - 9x + 20 = 0$. Since Jim copied b correctly, we can conclude that in the original quadratic equation, $b = -9$.

Similarly, since Vazz's roots are 2 and 4, his quadratic equation has the form $(x-2)(x-4) = x^2 - 6x + 8 = 0$. Since Vazz copied c correctly, then $c = 8$.

Thus, the original equation was $x^2 - 9x + 8 = 0$. Factoring, we obtain $(x-1)(x-8) = 0$. Therefore, the roots of the original equation are 1 and 8.

Also solved by CAO MINH QUANG, Nguyen Binh Khiem High School, Vinh Long, Vietnam; JACLYN CHANG, student, Western Canada High School, Calgary, AB; PETER CHIEN, student, Central Elgin Collegiate, St. Thomas, ON; IAN JUNE L. GARCES, Ateneo de Manila University, Quezon City, The Philippines; JOHAN GUNARDI, student, SMPK 4 BPK PENABUR, Jakarta, Indonesia; RICHARD I. HESS, Rancho Palos Verdes, CA, USA; RICARD PEIRÓ, IES "Abastos", Valencia, Spain; JOSÉ HERNÁNDEZ SANTIAGO, student, Universidad Tecnológica de la Mixteca, Oaxaca, Mexico; KUNAL SINGH, student, Kendriya Vidyalaya School, Shillong, India; BILLY SUANDITO, Palembang, Indonesia; LUYAN ZHONG-QIAO, Columbia International College, Hamilton, ON; and TITU ZVONARU, Comănești, Romania.

M339. *Proposed by the Mayhem Staff.*

(a) Determine the number of integers between 100 and 199, inclusive, which contain *exactly* two equal digits.

(b) An integer between 1 and 999 is chosen at random, with each integer being equally likely to be chosen. What is the probability that the integer has *exactly* two equal digits?