BOOK REVIEWS

John Grant McLoughlin

Math Made Visual
By Claudia Alsina and Roger B. Nelsen, published by the Math Association of America, 2006.
ISBN 0-88385-746-4, hardcover, 190 pages, US$49.95.
Reviewed by **J. Chris Fisher**, *University of Regina, Regina, SK*

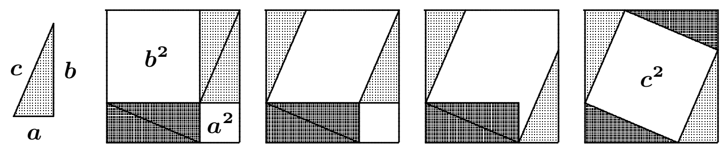
The book begins with a quotation from Martin Gardner:

> A dull proof can be supplemented by a geometric analogue so simple and beautiful that the truth of a theorem is almost seen at a glance.

These words sum up the authors' attitude toward proofs by pictures. Their goal here is to provide tools for devising one's own visual proofs. ***CRUX with MAYHEM*** readers may be familiar with the work of one of the authors, Roger B. Nelsen. In addition to having created many of his own "proofs without words", he has published two collections entitled *Proofs without Words*. Co-author Claudia Alsina likewise has good credentials.

The book comes in three parts. Part I consists of twenty chapters, each five or so pages in length and each describing a method to visualize some mathematical idea. The method is illustrated by means of several examples followed by a handful of exercises called *challenges*. The examples include some of the authors' favorite proofs without words, thankfully now with some words of explanation for those of us who find a proof without any words to be an annoyance.

Some of these proofs without words I have seen before, such as the proof of the Pythagorean Theorem, which comes in the chapter titled "Employing Isometry". You see a square containing four copies of an initial triangle whose sides are labeled a, b, c. The picture makes it quite clear that the total area of the white portion inside the large square remains unchanged as three of the four shaded triangles are translated to new positions. The authors might have included the caption $a^2 + b^2 = c^2$, but were content to leave that to the reader.

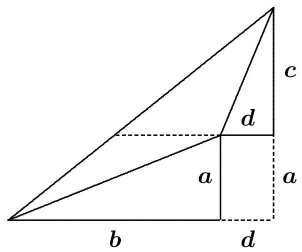


In the same chapter as this proof of the Pythagorean Theorem are four other worked examples with proofs based on rotations and translations. Among the four challenges that follow these results: Find the area of a convex octagon that is inscribed in a circle and has four consecutive sides of length 3 and the remaining four sides of length 2. Happily, Part III of the text consists of solutions or substantial hints to all the challenges of Part I.

(To solve the challenge just mentioned, divide the octagon into isosceles triangles from the centre of the circumcircle. Next rearrange the triangles, alternating the ones with base lengths 2 and 3. This new octagon can be inscribed in a square of side $3 + 2\sqrt{2}$, and whence, the area of the octagon is $(3 + 2\sqrt{2})^2 - 4(\sqrt{2} \cdot \sqrt{2})/2 = 13 + 12\sqrt{2}$.

The authors have done a fine service by collecting together this nice mathematics, thus making it more accessible. I had forgotten about the proof of the mediant property that appeared in *Mathematics Magazine* in 1990: For positive numbers a, b, c, d,

$$\frac{a}{b} < \frac{c}{d} \quad\Longrightarrow\quad \frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}.$$



The picture shows lines of slope $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{a+c}{b+d}$. It is clear which slopes are steeper. Although the obvious algebraic argument is no harder than this visual argument, the visualization would certainly make a more convincing classroom presentation.

The hundreds of visual arguments included by the authors are all elementary and transparent. However, many of these results have easier and more informative proofs. For example, a theorem of affine geometry affirms that *if a line intersects a hyperbola in points A and B and its asymptotes in A' and B', then segments AA' and BB' have the same length*. The authors restrict their theorem to a rectangular hyperbola and provide a clumsy verification using coordinates, an approach that seems out of place here. Probably most ***CRUX with MAYHEM*** readers would introduce affine coordinates with $A' = (1, 0)$, $B' = (0, \pm 1)$, and the origin as the intersection point of the asymptotes, then appeal to symmetry. (This would have been a good place to introduce the notion of an affine reflection, but the authors restrict their transformations to the more familiar Euclidean isometries.) We see in such examples, as well as in a general lack of references to original sources, that the book is simply a collection of items the authors have gathered over the years; it is not intended to be a work of careful scholarship.

For me, the only unsuccessful portion of the book is Part II, where the authors "... present some general pedagogical considerations concerning the development of visual thinking, practical approaches for making visualizations in the classroom and, in particular, the role that hands-on materials may play in this process." The sermon lasts 26 pages, but I saw nothing substantial; if the authors intended an important message, I certainly missed the point. Most of the other 150 pages contain a pleasant variety of interesting theorems, problems, and techniques.

I do not believe that the authors were successful in their goal of teaching readers how to devise their own neat visualizations. I wonder if such a skill can be taught. On the other hand, they have produced a nice collection of results and proofs that are worth knowing. I prefer this book to Nelsen's previous two collections because of its useful index and its explanations that accompany the visualizations.

Sensational Shape Problems & Other Puzzles
By Ivan Moscovich, published by Sterling Publishing Co., Inc., 2005
ISBN 1-4027-2347-4, soft cover, 128 pages, US$9.95.
Reviewed by ***Tanya Thompson****, Collingwood Collegiate Institute, Collingwood, ON.*

This book is one of twelve in Ivan Moscovich's *Mastermind Collection*. It presents recreational mathematics and puzzles visually in a pleasing way that entices the reader to play. As in all his books, the presentation is beautiful, and visual layouts help one to understand the essence of each problem.

This book presents a variety of puzzles (or "Thinkthings", as Moscovich calls them), from historical classics to innovative originals. Dissections, T–puzzles, tangrams, Pythagorean Theorem problems, packing puzzles, and geometrical paradoxes are all explored. The book provides answers for the problems, and historical facts are also presented where appropriate.

One of my favourite Thinkthings is a classic from the geometrical paradoxes, called "Disappearing Face Magic", consisting of a line of six faces of men in hats. When the picture is cut into two strips along an indicated black line, and the lower strip is slid to the left, one of the faces disappears. The reader is left to ask, "Which face disappeared?" Martin Gardner, one of the foremost advocates for recreational mathematics, has named this concept the *Principle of Concealed Distribution*. Moscovich mentions this principle in the book. In [1, pp. 117–128], Gardner explains it in greater detail.

As a high school mathematics teacher who loves recreational mathematics, I feel that *Sensational Shape Problems & Other Puzzles* is a great set of engaging problems. These problems are fun, as well as helpful in developing critical thinking and spatial skills necessary for curriculum-based problems. They could be used as warm-up activities or as investigations all their own. A wonderful thing about recreational problems is that they are appropriate for many different levels and abilities. Since basic mathematical skills are not always a requirement, many different learners can find success. With success comes confidence, and for many students confidence is key.

Martin Gardner once wrote, "A teacher of mathematics, no matter how much he loves his subject and how strong his desire to communicate, is perpetually faced with an overwhelming difficulty: How can he keep his students awake? . . . The best way, it has always seemed to me, to make mathematics interesting to students . . . is to approach it in a spirit of play" [2, p. xi]. This book does just that. The Thinkthings motivate students to play. The students will have fun problem-solving and become excited about mathematics. What could be better than that?

References

[1] Martin Gardner, *Mathematics, Magic and Mystery*, Dover Publications, Inc., New York, 1956.

[2] Martin Gardner, *1975 Mathematical Carnival: A New Round-up of Tantalizers and Puzzles from "Scientific American"*, Knopf Publishing Group, 1975.