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SYNOPSIS

39 Some Inversion Formulas for Sums of Quotients

by *Natalio H. Guersenzvaig and Michael Z. Spivey*

In this note the authors establish some formulas for certain sums of quotients of a positive integer n, which are closely related to an identity established by Préville-Ratelle in Problem M40 of the April 2003 issue of ***CRUX with MAYHEM***. They also establish some elementary facts that are not well known about quotients and remainders.

Their main result is the following theorem.

Theorem. Let n and k be any positive integers with $k \leq n$. Then

$$\sum_{d=1}^{k} \left\lfloor \frac{n}{d} \right\rfloor - \sum_{d=\lfloor \frac{n}{k} \rfloor + 1}^{n} \left\lfloor \frac{n}{d} \right\rfloor = k \left\lfloor \frac{n}{k} \right\rfloor .$$

Enjoy!

44 Problems: 3056, 3101–3113

This month's "free sample" is:

3101. *Proposed by K.R.S. Sastry, Bangalore, India.*

The two distinct cevians AP and AQ of $\triangle ABC$ satisfy the equation $AQ^2 = AP^2 + |AC - AB|^2$.

(a) If $BP = CQ$, show that AP bisects $\angle BAC$.

(b)★ If AP bisects $\angle BAC$, prove or disprove that $BP = CQ$.

. . . .

3101. *Proposé par K.R.S. Sastry, Bangalore, Inde.*

Les deux céviennes distinctes AP et AQ d'un triangle ABC satisfont l'équation $AQ^2 = AP^2 + |AC - AB|^2$.

(a) Si $BP = CQ$, montrer que AP est une bissectrice de l'angle BAC.

(b)★ Si AP est une bissectrice de l'angle BAC, démontrer ou réfuter l'égalité $BP = CQ$.

50 Solutions: 2923, 2984, 3001–3007