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SYNOPSIS

129 Skoliad: No. 85 *Robert Bilinski*

- 4th Annual CNU Regional High School Mathematics Contest 2003 (4e Concours Annuel CNU Régional de Mathématique du Secondaire 2003)
- Solutions to the 2004 BC Colleges High School Mathematics Contest, Junior and Senior Final Rounds

138 Mathematical Mayhem

138 Mayhem Problems: M188–M193

M188. *Proposed by Charalampos Stergiou, Chalkida, Greece.*

Consider triangle ABC in which $\angle B = \angle C = 35^\circ$. In the interior of the triangle we take a point M such that $\angle MBC = 25^\circ$ and $\angle MCB = 30^\circ$. Prove, without trigonometry, that $\angle AMC = 150^\circ$.

M189. *Proposed by Mihály Bencze, Brasov, Romania.*

Find all real solutions of the following system of equations:

$$\begin{aligned} x + \sqrt{x^2+1} &= 10^{y-x}\,, \\ y + \sqrt{y^2+1} &= 10^{z-y}\,, \\ z + \sqrt{z^2+1} &= 10^{x-z}\,. \end{aligned}$$

M190. *Proposed by Li Zhou, Polk Community College, Winter Haven, FL, USA.*

Given any three points in a unit square, show that a pair of them must be no further apart than $\sqrt{6}-\sqrt{2}$.

M191. *Proposed by the Mayhem Staff.*

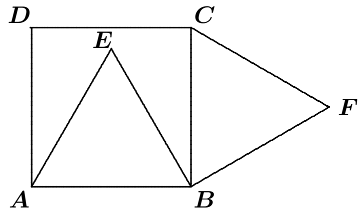
The surface areas of the six faces of a rectangular prism (box) are 1254, 1254, 770, 770, 1995, and 1995 cm^2. Determine the volume of the prism.

M192. *Proposed by Victor Oxman, Western Galilee College, Israel.*

In triangles $A_1B_1C_1$ and $A_2B_2C_2$, we are given that $A_1C_1 = A_2C_2$, that the medians B_1M_1 and B_2M_2 are equal, and that the bisectors A_1D_1 and A_2D_2 are equal. Prove that the triangles are congruent.

M193. *Proposed by Robert Bilinski, Outremont, QC.*

On square $ABCD$, an equilateral triangle ABE is constructed internally and an equilateral triangle BCF is constructed externally. Prove that the points D, E, and F are collinear.



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M188. *Proposé par Charalampos Stergiou, Chalkida, Grèce.*

Dans un triangle ABC les angles B et C mesurent $35°$. On choisit un point M à l'intérieur du triangle de sorte que les angles MBC et MCB valent respectivement $25°$ et $30°$. Sans trigonométrie, montrer que l'angle AMC vaut $150°$.

M189. *Proposé par Mihály Bencze, Brasov, Roumanie.*

Trouver toutes les solutions réelles du système d'équations suivant :

$$\begin{aligned} x+\sqrt{x^2+1} &= 10^{y-x}\,, \\ y+\sqrt{y^2+1} &= 10^{z-y}\,, \\ z+\sqrt{z^2+1} &= 10^{x-z}\,. \end{aligned}$$

M190. *Proposé par Li Zhou, Polk Community College, Winter Haven, FL, USA.*

Etant donné trois points dans un carré unité, montrer qu'une paire d'entre eux ne peuvent être distants de plus de $\sqrt{6}-\sqrt{2}$.

M191. *Proposé par Équipe de Mayhem.*

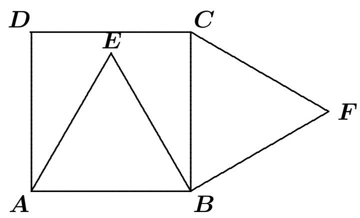
Les aires des six faces d'un prisme rectangulaire (une boîte) valent 1254, 1254, 770, 770, 1995 et 1995 cm^2. Calculer le volume du prisme.

M192. *Proposé par Victor Oxman, Western Galilee College, Israël.*

Montrer que les triangles $A_1B_1C_1$ et $A_2B_2C_2$ sont congruents, sachant que $A_1C_1 = A_2C_2$, que les médianes B_1M_1 et B_2M_2 ainsi que les bissectrices A_1D_1 et A_2D_2 sont égales.

M193. *Proposé par Robert Bilinski, Outremont, QC.*

On trace à l'intérieur du carré $ABCD$ un triangle équilatéral ABE et à l'extérieur un triangle équilatéral BCF. Montrer que les points D, E et F sont alignés.



140 Mayhem Solutions: M130–M132

144 Problem of the Month *Ian VanderBurgh*

146 Pólya's Paragon: Fun With Numbers (Part 3)

148 Iterating Möbius Functions with Rational Coefficients, Part II
by *Kun-Chieh Wang*

A *Möbius function* is a function of the form

$$f(z) = \frac{az+b}{cz+d},$$

where z is a complex variable and the coefficients a, b, c, and d are complex numbers such that $ad \neq bc$.

In this paper, the author completes the determination of all the possible periods of a periodic sequence of functions obtained by iterating a Möbius function with rational coefficients.

Enjoy!

150 The Olympiad Corner: No. 245 *R.E. Woodrow*

Featuring the XX Colombian Mathematical Olympiad 2001; 53rd Polish Mathematical Olympiad 2001–02; 2nd Czech-Polish-Slovak Mathematical Competition 2002; a comment on problem #23 from the St. Petersburg Contests 1965–1984; and readers' solutions to some of the problems from

- the 2000 Bulgarian Mathematical Olympiad;
- the 2000 Belarusian Mathematical Olympiad;
- the 2000 Taiwanese Mathematical Olympiad.

167 Book Review *John Grant McLoughlin*

167 *TriMathlon: A Workout Beyond the School Curriculum*
by Judith D. Sally and Paul J. Sally, Jr.
Reviewed by Anne Izydorczak

168 The Diagonal Points of a Cyclic Quadrangle

by *Christopher J. Bradley*

Let $ABCD$ be a cyclic quadrangle in which there are no parallel sides, such that the diagonals AC and BD meet at E, and AB and CD meet at F, AD and BC meet at G. Then EFG is the diagonal-point triangle, which is self-conjugate.

A number of results are established about various triangle centres for the triangles in the set $\{AFG, BFG, CFG, DFG\}$ and for those in the set $\{ACF, BDF, ACG, BDG\}$.

Enjoy!

173 Problems: 3007, 3026–3038

This month's "free sample" is:

3027. *Proposed by Geoffrey A. Kandall, Hamden, CT, USA.*

Let $ABCD$ be any quadrilateral, and let M be the mid-point of AB. On the sides CB, DC, and AD, equilateral triangles CBE, DCF, and ADG are constructed externally. Let N be the mid-point of EF and P be the mid-point of FG.

Prove that $\triangle MNP$ is equilateral.

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3027. *Proposé par Geoffrey A. Kandall, Hamden, CT, USA.*

Soit $ABCD$ un quadrilatère quelconque, et soit M le point milieu de AB. Sur les côtés CB, DC, and AD, on construit extérieurement les triangles équilatéraux CBE, DCF et ADG. Soit N le point milieu de EF et P le point milieu de FG.

Montrer que le triangle MNP est équilatéral.

179 Solutions: 2927–2938