Mayhem Problems

Please send your solutions to the problems in this edition by **1 September 2005**. *Solutions received after this date will only be considered if there is time before publication of the solutions.*

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, and 7, English will precede French, and in issues 2, 4, 6, and 8, French will precede English.

The editor thanks Jean-Marc Terrier and Martin Goldstein of the University of Montreal for translations of the problems.

M188. *Proposed by Charalampos Stergiou, Chalkida, Greece.*

Consider triangle ABC in which $\angle B = \angle C = 35^\circ$. In the interior of the triangle we take a point M such that $\angle MBC = 25^\circ$ and $\angle MCB = 30^\circ$. Prove, without trigonometry, that $\angle AMC = 150^\circ$.

M189. *Proposed by Mihály Bencze, Brasov, Romania.*

Find all real solutions of the following system of equations:

$$\begin{aligned} x + \sqrt{x^2+1} &= 10^{y-x}\,, \\ y + \sqrt{y^2+1} &= 10^{z-y}\,, \\ z + \sqrt{z^2+1} &= 10^{x-z}\,. \end{aligned}$$

M190. *Proposed by Li Zhou, Polk Community College, Winter Haven, FL, USA.*

Given any three points in a unit square, show that a pair of them must be no further apart than $\sqrt{6} - \sqrt{2}$.

M191. *Proposed by the Mayhem Staff.*

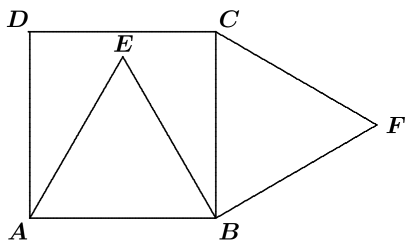
The surface areas of the six faces of a rectangular prism (box) are 1254, 1254, 770, 770, 1995, and 1995 cm^2. Determine the volume of the prism.

M192. *Proposed by Victor Oxman, Western Galilee College, Israel.*

In triangles $A_1B_1C_1$ and $A_2B_2C_2$, we are given that $A_1C_1 = A_2C_2$, that the medians B_1M_1 and B_2M_2 are equal, and that the bisectors A_1D_1 and A_2D_2 are equal. Prove that the triangles are congruent.

M193. *Proposed by Robert Bilinski, Outremont, QC.*

On square $ABCD$, an equilateral triangle ABE is constructed internally and an equilateral triangle BCF is constructed externally. Prove that the points D, E, and F are collinear.



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M188. *Proposé par Charalampos Stergiou, Chalkida, Grèce.*

Dans un triangle ABC les angles B et C mesurent $35°$. On choisit un point M à l'intérieur du triangle de sorte que les angles MBC et MCB valent respectivement $25°$ et $30°$. Sans trigonométrie, montrer que l'angle AMC vaut $150°$.

M189. *Proposé par Mihály Bencze, Brasov, Roumanie.*

Trouver toutes les solutions réelles du système d'équations suivant :

$$\begin{aligned} x+\sqrt{x^2+1} &= 10^{y-x}\,, \\ y+\sqrt{y^2+1} &= 10^{z-y}\,, \\ z+\sqrt{z^2+1} &= 10^{x-z}\,. \end{aligned}$$

M190. *Proposé par Li Zhou, Polk Community College, Winter Haven, FL, USA.*

Etant donné trois points dans un carré unité, montrer qu'une paire d'entre eux ne peuvent être distants de plus de $\sqrt{6}-\sqrt{2}$.

M191. *Proposé par Équipe de Mayhem.*

Les aires des six faces d'un prisme rectangulaire (une boîte) valent 1254, 1254, 770, 770, 1995 et 1995 cm^2. Calculer le volume du prisme.

M192. *Proposé par Victor Oxman, Western Galilee College, Israël.*

Montrer que les triangles $A_1B_1C_1$ et $A_2B_2C_2$ sont congruents, sachant que $A_1C_1 = A_2C_2$, que les médianes B_1M_1 et B_2M_2 ainsi que les bissectrices A_1D_1 et A_2D_2 sont égales.

M193. *Proposé par Robert Bilinski, Outremont, QC.*

On trace à l'intérieur du carré $ABCD$ un triangle équilatéral ABE et à l'extérieur un triangle équilatéral BCF. Montrer que les points D, E et F sont alignés.