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SYNOPSIS

257 Skoliad: No. 71 *Shawn Godin*

2003 Fryer Contest

Nineteenth W.J. Blundon Mathematics Contest (2002)

Solutions to the 2^{nd} Junior Balkan Mathematical Olympiad (1998)

Solutions to the 2002 British Columbia Colleges Junior High School Mathematics Contest, Preliminary Round

265 Mathematical Mayhem

265 Mayhem Problems: M101–M106

Here is a "free sample":

M105. *Proposed by Andrew Critch, Clarenville High School, Clarenville, NL.*

Suppose that the roots of $P(x) = x^3 - 2kx^2 - 3x^2 + hx - 4$ are distinct, and that $P(k) = P(k+1) = 0$. Determine the value of h.

. . . .

On suppose que les racines de $P(x) = x^3 - 2kx^2 - 3x^2 + hx - 4$ sont distinctes, et que $P(k) = P(k+1) = 0$. Trouver la valeur de h.

267 Mayhem Solutions: M51–M56

272 Pólya's Paragon *Paul Ottaway*

275 Binomial Inversion: Two Proofs and an Application to Derangements *Heba Hathout*

279 The Olympiad Corner: No. 231 *R.E. Woodrow*

Featuring the 2000 International Mathematical Olympiad Shortlisted Problems; and readers' solutions to some of the problems of

- the Vietnamese Mathematical Competition 1997;
- the Turkey Team Selection Examination for the 38^{th} IMO 1997;
- the Chilean Mathematical Olympiads 1994–95;

The inequality in the title has recently been established by Bhatia and Davis using calculus. This note proves the result without using calculus.

Let k be any field, and let $f(X)$ be an arbitrary polynomial of $k[X]$ which is irreducible in $k[X]$. A result of Wahlen-Capelli establishes necessary and sufficient conditions for the irreducibility of $f(g(X))$ in $k[X]$, where $g(X)$ is any polynomial of $k[X]$. The proof of this result is not elementary because it uses the theory of field extensions.

The author establishes necessary and sufficient conditions for the reducibility of $f(X^2)$ in $Z[X]$, where Z denotes an arbitrary unique factorization domain. As an immediate consequence we obtain a simple sufficient condition for the irreducibility of $f(X^2)$ in $Z[X]$.

Read on!

This month's "free sample" is:

2854. *Proposed by Toshio Seimiya, Kawasaki, Japan.*

Suppose that M and N are the mid-points of the sides AB and CD of quadrilateral $ABCD$, respectively.

Prove that $AN^2 + DM^2 + BC^2 = BN^2 + CM^2 + AD^2$.

. . . .

Supposons que M et N sont les points milieu respectifs des côtes AB et CD d'un quadrilatère $ABCD$.

Montrer que $AN^2 + DM^2 + BC^2 = BN^2 + CM^2 + AD^2$.