Mayhem Problems

Proposals and solutions may be sent to **Mathematical Mayhem, 2191 Saturn Crescent, Orleans, Ontario, K4A 3T6** or emailed to

`mayhem-editors@cms.math.ca`

Please include in all correspondence your name, school, grade, city, province or state, and country. We are especially looking for solutions from high school students. Please send your solutions to the problems in this edition by *1 March 2004*. Solutions received after this time will be considered only if there is time before publication of the solutions.

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, and 7, English will precede French, and in issues 2, 4, 6, and 8, French will precede English.

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M101. *Proposed by the Mayhem Staff.*

Find the smallest value of k such that $k!$ ends with 100 zeros. [*Note:* $k! = k(k-1)(k-2)\cdots(3)(2)(1)$.]

. . . .

Trouver la plus petite valeur de k telle que $k!$ finisse avec 100 zéros. [*Note :* $k! = k(k-1)(k-2)\cdots(3)(2)(1)$.]

M102. *Proposed by Bruce Shawyer, Memorial University of Newfoundland, St. John's, NL.*

Suppose that $ABCD$ is a parallelogram and that G_A, G_B, G_C, and G_D are the centroids of $\triangle BCD$, $\triangle ACD$, $\triangle ABD$, and $\triangle ABC$, respectively.

Prove that:

1. $G_AG_BG_CG_D$ is a parallelogram;
2. $[G_AG_BG_CG_D] = \frac{1}{9}[ABCD]$, where $[ABCD]$ is the area of $ABCD$.

. . . .

Dans un parallélogramme $ABCD$ on suppose que G_A, G_B, G_C et G_D sont les centres de gravité respectifs des triangles BCD, ACD, ABD et ABC.

Montrer que :

1. $G_AG_BG_CG_D$ est un parallélogramme ;
2. $[G_AG_BG_CG_D] = \frac{1}{9}[ABCD]$, où $[ABCD]$ désigne l'aire de $ABCD$.

M103. *Proposed by the Mayhem Staff.*
Solve for n:

$$100^{1/n} \times 100^{2/n} \times 100^{3/n} \times \cdots \times 100^{2003/n} = 1000\,.$$

. . . .

Résoudre par rapport à n :

$$100^{1/n} \times 100^{2/n} \times 100^{3/n} \times \cdots \times 100^{2003/n} = 1000\,.$$

M104. *Proposed by Bruce Shawyer, Memorial University of Newfoundland, St. John's, NL.*

Suppose that $ABCD$ is a parallelogram and that O_A, O_B, O_C, and O_D are the circumcentres of $\triangle BCD$, $\triangle ACD$, $\triangle ABD$, and $\triangle ABC$, respectively.

Prove that:

1. $O_AO_BO_CO_D$ is a parallelogram;
2. parallelograms $ABCD$ and $O_AO_BO_CO_D$ are similar;
3. AO_BCO_D is a parallelogram;
4. O_ABO_CD is a parallelogram;
5. parallelograms AO_BCO_D and O_ABO_CD are similar.

. . . .

Dans un parallélogramme $ABCD$ on suppose respectivement que O_A, O_B, O_C and O_D sont les centres des cercles circonscrits des triangles BCD, ACD, ABD and ABC.

Montrer que :

1. $O_AO_BO_CO_D$ est un parallélogramme ;
2. les parallélogrammes $ABCD$ et $O_AO_BO_CO_D$ sont semblables ;
3. AO_BCO_D est un parallélogramme ;
4. O_ABO_CD est un parallélogramme ;
5. les parallélogrammes AO_BCO_D et O_ABO_CD sont semblables.

M105. *Proposed by Andrew Critch, Clarenville High School, Clarenville, NL.*

Suppose that the roots of $P(x) = x^3 - 2kx^2 - 3x^2 + hx - 4$ are distinct, and that $P(k) = P(k+1) = 0$. Determine the value of h.

. . . .

On suppose que les racines de $P(x) = x^3 - 2kx^2 - 3x^2 + hx - 4$ sont distinctes, et que $P(k) = P(k+1) = 0$. Trouver la valeur de h.

M106. *Proposed by the Mayhem Staff.*

A 4 by 4 square has an area of 16 square units and a perimeter of 16 units. That is, the area and perimeter are numerically equivalent (ignoring units of measurement). Are there any other rectangles with integral dimensions that share this property? If possible, show that you have found all such examples.

Un carré de 4 par 4 a une aire de 16 unités carrées et un périmètre de 16 unités. Autrement dit, l'aire et le périmètre sont numériquement équivalents (si on laisse tomber les unités). Y a-t-il d'autres rectangles de dimensions entières possédant cette propriété ? Si possible, montrez que vous les avez tous trouvés.