BOOK REVIEWS

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Inverse problems - Activities for Undergraduates
by Charles W. Groetsch, published by The Mathematical Association of America, 1999
ISBN# 0-88385-716-2, softcover, 222 + xii pages, US$26.00
Reviewed by ***Edward Vrscay****, University of Waterloo, Waterloo, Ontario.*

In the preface, the author begins by writing, "This is not a textbook. Nor is it a survey of elementary inverse problems. It is a personal miscellany of activities related to inverse problems that is meant to enrich, and perhaps enliven, the teaching of mathematics in the first two undergraduate years."

Indeed, this "miscellany", written by a specialist in inverse and ill-posed problems, is a delightful gem that could be used at various places and levels of undergraduate school. As the author points out in his first chapter, *Introduction to Inverse Problems*, undergraduate training in mathematics and science is generally dominated by *direct* problems—those in which a student is given enough information to carry out a well-defined procedure that yields a unique solution. In other words, the student is given a "model" (process) along with a "cause" (input) and is then expected to find the "effect" (output). But what about the other side of the coin? For example, if we have a model K and an effect y, can we find the cause x? Or if we know the cause x and effect y, can we come up with the model K? These are *inverse* problems. Not surprisingly, inverse problems constitute a significant chapter in the history of science, engineering and mathematics.

As a very simple example, consider the linear interpolation problem, viewed by the author as the oldest problem in mathematics. The direct problem is to calculate the values of a linear function. The inverse problem is to determine a linear function from some data points (x_i, y_i).

The author briefly discusses a number of classic inverse problems from history, including "Archimedes' Bath" (technically, "nondestructive evaluation"), the two-body Kepler problem and, more recently, computed tomography (Radon transform). These very readable vignettes should impress upon the reader the importance of inverse problems in the history of mathematics and science, which should be reason enough to include them in undergraduate education. However, there is a more important point made by the author in the section entitled *Why Teach Inverse Problems*, namely, that teaching inverse problems nourishes a habit of "inverse thinking" in students. Looking only at the direct problem is to miss much of the whole picture. Teaching only direct methods is, in fact, intellectually limiting. (This reviewer agrees wholeheartedly with the author's sentiments.)

The next four chapters of the book are devoted to inverse problems in precalculus, calculus, differential equations and linear algebra, respectively. Each chapter has six "modules" that consist of: (i) an introduction for the instructor (giving the course level of the problem(s) of concern, goals, mathematical and scientific background required of the student, and necessary hardware/software), (ii) Activities (exercises and computations), and (iii) Notes and Further Reading. Answers and advice for selected problems are found in Appendix A. Appendix B contains source codes for MATLAB routines that can be used for some of the computations in the book. (These source codes can also be downloaded from the author's website.)

For example, the module entitled *Shape Up!* in Chapter 2, (*Inverse Problems in Calculus*), examines direct and inverse problems for the following drainage scenario: A vessel is formed by revolving a curve $x = f(y)$ about the y-axis. For a given initial water depth $y > 0$, let $T(y)$ be the time necessary for the vessel to drain through an orifice of cross-sectional area a at its base. This module examines the relationship between the "drain-time" function T and f. (One will need Torricelli's Law, discussed in the earlier module *A Little Squirt*.) The direct problem is to find T given f: A number of functions f are suggested in the Activities section. Students are also asked to ascertain some mathematical properties of T. The inverse problem is to find f given T. This includes the classic *clepsydra* or water-clock problem (finding f so that $T(y)$ is linear in y).

There are also opportunities to explore more realistic aspects of this inverse problem, for example, when the drain-time function T is not given in closed form, but rather in the form of data points (for example, stopwatch experiments). The student is asked to explore some approximation schemes (numerical integration and differentiation) using the MATLAB programs provided. The accuracy of these schemes is to be examined along with the effects of noise in the data.

The chapter on inverse problems in linear algebra includes problems in elementary tomography, gravimetry and inverse vibration. The mathematical methods used there include projections onto hyperplanes, matrix and generalized inverses, and eigenvalue analysis. For example, the module entitled "L'ART Pour L'Art" examines the *algebraic reconstruction technique* (ART) algorithm, which uses successive orthogonal projections to approximate solutions of linear equations. A simple tomography problem is then considered.

You might be able to find some of the inverse problems that are treated in this book scattered throughout some standard first- and second-year calculus and linear algebra texts. However, you probably won't find these problems treated with the care and possibly the depth that they receive in this book. Nor will you find anywhere else such a comprehensive collection of "inverse thinking" under one cover. This book would serve as a valuable resource not only in Year 1 and 2 Calculus and Linear Algebra courses but also

in courses on differential equations and Newtonian mechanics.

My only complaint about this book is concerned with the numbering of sections in Appendix A, *Selected Answers and Advice*: Section $A.m.n$ in the Appendix corresponds to module $(m-1).n$ in the main text. I find that I must almost always go back to the Table of Contents to return from a solution to the appropriate module in the main text. I would hope that future editions of the book would at least add the starting page of the module to the subtitles of the Appendix; for example *Slip Sliding Away* (p. 96). However, this is but a minor blemish in what is a unique and highly recommended book, clearly a labour of love by an expert in the field.

Challenging Brainteasers
by Bernardo Recamán Santos, published by Sterling Publishing Co., Inc., 1997
ISBN 0-8069-2877-8, softcover, 93 pages, US$9.95
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This book is a compilation of mathematical games and puzzles which the author has either created or collected over the years. He divides the problems into four categories: Arithmetical Puzzles, Geometrical Puzzles, Logical Puzzles, and Algebraic Puzzles. The puzzles range from simple algebraic word problems to some relatively difficult combinatorial problems.

Each section has 14 or 15 questions. Many of the problems are probably familiar to readers of such articles. In the foreword, the author explains that most of the problems are ones that he has collected, and only a few are original. The solutions provided for the problems range from simple answers to more complete discussions of the problem. In some cases, the author suggests variations on the problem, or the problems introduce unusual mathematical terms such as "exotic numbers". The book is probably suitable as a resource for high school or senior elementary school teachers. Teachers can use the problems in their classroom as enrichment, or as a catalyst to start discussing other mathematical concepts.

Although educators can always find a place on their bookshelf for this type of compilation, *Challenging Brainteasers* is not an extraordinary collection of problems and solutions. The solutions section could have been expanded significantly. Ideally it would include more problem-solving techniques, more variations for the problems, and more complete solutions rather than just answers. As a set of classic math problems this book is adequate, but if you are looking for more you will not find it here.