BOOK REVIEWS

John Grant McLoughlin

The Golden Section
by Hans Walser, as translated from the original German by Peter Hilton, with the assistance of Jean Pedersen, published by the Mathematical Association of America (Spectrum Series), 2001,
ISBN 0-88385-534-8, softcover, 136 + pages, US$26.95.
Reviewed by **Walter M. Reid**, *University of Wisconsin-Eau Claire, Eau Claire, Wisconsin.*

This book presents many perspectives on the golden section (golden ratio), keeping a clear focus on its topic. In independent chapters, the book presents the golden section first by examples and definition, then through fractals, golden (constructive) geometry, paper folds (origami) and cuts, number (Fibonacci) sequences, regular and semi-regular solids, and finishes with a chapter on graph intersections, extremal values, and probabilities of winning games.

I have often wondered about the golden ratio and its origin, and yet had never happened upon a concise and satisfactory presentation. Therefore, to begin with, the book's topic caught my attention and interest. It is also an attractive book. Its cover features Leonardo da Vinci's *Mona Lisa* with facial and other golden rectangles superimposed, one of which serves to box in the defining equation for the golden ratio. The book proper then presents a very clean typographical style with a wealth of fascinating graphics and illustrations throughout. The clean look of the text is due in part to a somewhat terse narrative. Few words are wasted. Each section begins with a brief argument or explanation of a relationship, graph or diagram, followed by questions inviting the reader to continue developing the material in the same vein. The book is inviting to pick up and thumb through.

The book begins quickly with several examples of graphs demonstrating the ratio. As it turns out, the book returns later to develop and explore each of those introductory examples. The definition of the golden section appears (boxed in) immediately on page 2:

"We say that a line-segment is divided in the ***ratio of the Golden Section***, or the ***Golden Ratio***, if the larger segment is related to the smaller exactly as the larger segment is related to the whole segment."

This definition leads directly to the defining (cover) equation:

$$\frac{x}{1-x} = \frac{1}{x},$$

which, in turn, leads to the golden ratio $\tau = \frac{\sqrt{5}+1}{2} \approx 1.61803$ and its reciprocal $\rho = \frac{\sqrt{5}-1}{2} \approx 0.61803$.

My favourite chapters were Chapter 1, the introductory chapter; Chapter 2 on fractals, where the development of fractal dimension intrigued me; Chapter 5 on number (Fibonacci and generalized Fibonacci) sequences with an intriguing development of the family tree of a drone (bee); and Chapter 7, the last chapter, on intersections of graphs, extremal values, and probabilities of winning games.

Chapters 3 and 5 are the longest. Chapter 3 on golden geometry begins with a challenging argument. It establishes the construction of numbers whose decimal representations differ from those of their reciprocals by a whole number n, thus sharing with them the same fractional parts. This generalizes the case of the Golden Ratio, where $n = 1$. Later in this chapter, the topics of equal areas of ellipses and circles and the geometry of a musical cassette are also fascinating.

The questions, 80 of them in all, begin in the second chapter, on fractals, and then continue (not uniformly) throughout the rest of the book. Chapter 7 finishes with a burst of them, the last 20 questions appearing in five of the last pages of the book. The questions are, for the most part, presented in a casual and inviting way. Most of them are interesting but many are non-routine. All the answers, but not the solutions, are offered at the end of the book. While the questions are usually posed in a casual fashion, it is often unclear what the essence of a question is. I often found myself reading the answer for guidance and then returning to the question. Some of the answers contained surprises, for example, topological equivalence, which was not found in the index.

Though the topics are interesting and the variety of applications is enticing, their development seems somewhat uneven. The more algebraic arguments seem smoother than many of the geometric arguments. While this perception most likely reflects my own preferences, it struck me that the general audience for which the Spectrum Series reportedly is written, might share similar reactions. Furthermore, some arguments included geometric series, limits, elementary matrix theory and eigenvalues, and mathematical induction. While the book lists no prerequisites, having a rudimentary grasp of elementary calculus, linear algebra, number theory, topology, plane and solid geometry would serve the reader well. The high school graduate may find this book a challenge to read, the college graduate in mathematics with a strong geometry background should find it a pleasure to read, and even the geometer might find some fascinating nuggets awaiting. I personally enjoyed the book, and yes, it did satisfy my curiosity about the golden ratio.

On a final note, I must say that the book seemed free of errors in the parts I read carefully.