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SYNOPSIS

287 Introduction

Introducing the next Editor-in-Chief, Jim Totten

289 The Olympiad Corner: No. 223 *R.E. Woodrow*

Featuring a continuation of the problems from the St. Petersburg Contests 1965–1984; a reader's response to a question we posed when giving solutions to problem 5 of the Georg Mohr Konkurrencen I Matematik 1996; a missing solution when we gave responses to the Second Round of the 13$^{\text{th}}$ Iranian Mathematical Olympiad 1996; readers' solutions to problems of the XXXIII Spanish Mathematical Olympiad 1996–97; and readers' solutions to some of the problems of the 20$^{\text{th}}$ Austrian-Polish Mathematical Competition 1997.

306 Book Reviews *John Grant McLoughlin*

Mathematical Reflections: In a Room with Many Mirrors
by Peter Hilton, Derek Holton and Jean Pedersen
Mathematical Vistas: From a Room with Many Windows
by Peter Hilton, Derek Holton and Jean Pedersen

308 Some generalizations of an inequality from IMO 2001

Oleg Mushkarov and Nikolai Nikolov

The purpose of this paper is to consider some natural generalizations of Problem 2 from IMO 2001 which states:

Prove that

$$\frac{a}{\sqrt{a^2+8bc}}+\frac{b}{\sqrt{b^2+8ac}}+\frac{c}{\sqrt{c^2+8ab}} \;\geq\; 1\,,$$

where a, b and c are arbitrary positive numbers.

Many different proofs of this inequality were given during the Olympiad and it was also shown by the first author that

$$\frac{a}{\sqrt{a^2+\lambda bc}}+\frac{b}{\sqrt{b^2+\lambda ac}}+\frac{c}{\sqrt{c^2+\lambda ab}} \;\geq\; \frac{3}{\sqrt{1+\lambda}}$$

for arbitrary a, b, $c > 0$ and $\lambda \geq 8$. It is easy to see that the latter inequality is not true for $0 < \lambda < 8$. Moreover, it can be shown that in this case

$$\frac{a}{\sqrt{a^2 + \lambda bc}} + \frac{b}{\sqrt{b^2 + \lambda ac}} + \frac{c}{\sqrt{c^2 + \lambda ab}} > 1 ,$$

and the lower bound is sharp.

We now prove a general inequality that encompasses all of these results.

Read on!

This month's "free sample" is dedicated to Professor Jordi Dou, belatedly for his 90$^{\text{th}}$ birthday (dédiés tardivement au Professeur Jordi Dou, pour son 90-ième anniversaire):

2753. *Proposed by Mikhail Kotchetov, Memorial University of Newfoundland, St. John's, Newfoundland and Labrador.*

Consider two circles, Γ_1 and Γ_2, centres O_1 and O_2, respectively, of different radii.

The two common tangents, t_1 and t_2, that do not intersect the line segment O_1O_2 meet at Q. A common tangent, t_c that does intersect the line segment O_1O_2 meets the tangents t_1 and t_2 at E_1 and E_2, respectively.

Let P be the mid-point of the line segment O_1O_2.

Prove that P, Q, E_1 and E_2 are concyclic.

Considérons deux cercles Γ_1 et Γ_2, de centres respectifs O_1 and O_2 et de rayons différents.

Soit Q l'intersection des deux tangentes communes t_1 et t_2 qui ne coupent pas le segment O_1O_2. Désignons par t_c une tangente commune coupant O_1O_2, et par E_1 et E_2 les points d'intersection de t_c avec t_1 et t_2, respectivement.

Soit P le point milieu du segment O_1O_2.

Montrer que P, Q, E_1 et E_2 sont sur un même cercle.