

BOOK REVIEWS

JOHN GRANT McLOUGHLIN

Mathematical Reflections: In a Room with Many Mirrors, by Peter Hilton, Derek Holton and Jean Pedersen, published by Springer-Verlag, 1997, ISBN 0-387-94770-1, hardcover, 351 pages, 138 illus., US\$44.95.

Mathematical Vistas: From a Room with Many Windows, by Peter Hilton, Derek Holton and Jean Pedersen, published by Springer-Verlag, 2002, ISBN 0-387-95064-8, hardcover, 350 pages, 162 illus., US\$59.00.

Reviewed by **G. Eric Moorhouse**, University of Wyoming, Laramie, WY, USA.

Contributions to the literature of recreational mathematics have proliferated in recent years, although most works in this genre offer less depth than breadth.

These two books—we will call them *Reflections* and *Vistas*—help to fill that void by presenting a limited selection of mathematical topics of general interest (some of which I have listed below) in sufficient detail to satisfy the serious undergraduate student's penchant for completeness and concreteness, as well as the instructor's regard for correctness. Although technically demanding upon our eyes and fingers as we verify the mechanical steps throughout, at least nothing has been swept under the rug in these *Rooms*.

But readers will find in *Reflections* and *Vistas* more than another couple of collections of popular tidbits. They in fact offer us an informal glimpse into the minds of three professional mathematicians. Indeed, their first stated purpose is to realise what the authors call the *basic principle of mathematical instruction*, asserting that “mathematics must be taught so that students comprehend how and why mathematics is done by those who do it successfully.” Their second (and equally important) stated purpose, is to attract its readers, by impressing them with the sheer delight of mathematics, to the engagement of mathematical activity. (That is *activity*, not just *study*, a distinction maintained by the authors throughout.) These goals are admirably achieved by a lively selection of mathematical topics delivered in a casual writing style, in which technical arguments are richly interspersed with comments on the guiding principles of mathematical investigation (such as looking for symmetry when possible, favouring conceptual proofs over less illuminating proofs, etc.)

These concrete principles—the authors' advice on how mathematics should be “made”, “done”, written and taught (thereby transcending the mere “reading” and “learning” of mathematics)—are then listed and explained in the last chapter of *Reflections*. My favourite of these, “Be Optimistic!”, encourages us to approach all mathematical endeavours, everything from

homework to independent research, with the faith that it all makes sense. Observance of this principle will, in particular, encourage us to state mathematical conjectures in the greatest generality possible.

Reflections and *Vistas* are aimed at “secondary students of mathematics, undergraduate students of mathematics, [and] adults wishing to update and upgrade their mathematical competence.” They hit very near the intended level, I think. The reader may approach these books with little more than some appreciation for beauty in mathematics, and moderate patience and modest skills with algebraic manipulation. Groups, graphs, and equivalence relations are introduced as the need arises. Although no comparable preparation is supplied within for the use of mathematical induction, appearances of induction are limited to a couple instances clearly labeled as supplements for the more experienced reader (for example, the “difficult argument” of *Reflections*, pp. 200–202).

The mathematical content of *Reflections* and *Vistas* includes topics not only with strong popular appeal, but also well within the areas of mathematical expertise of the authors (often in fact from areas in which the authors have published research papers individually or jointly). These topics include

- Fibonacci and Lucas numbers^{*RV*};
- Paper-Folding Constructions of Polygons and Polyhedra^{*RV*};
- Graph Theory^{*V*}: Planarity, Chromatic Numbers, Ramsey Theory^{*V*};
- Infinite Cardinals^{*R*};
- Fractals and Dimension^{*R*};
- Binomial and Multinomial Coefficients^{*RV*}, Catalan numbers^{*V*};
- Paradoxes^{*V*};
- Symmetry^{*RV*}, Groups and Polya Counting^{*V*};

and much more. (The superscripts *R*, *V* and *RV* indicate topics found in *Reflections*, *Vistas* or both. Note the significant overlap between these two books, sometimes bordering on redundancy.) Not much harm would result from reading the chapters in a different order than that in which they appear, or from omitting certain chapters entirely. Yet the presentation is nicely unified, in both style and content—surprisingly so, considering the triple authorship. For example, their study of folded paper models provides concrete examples of discrete symmetry groups, but also (more surprisingly) is shown to relate to their earlier work on Fibonacci numbers (since in both settings one finds an integer sequence for which a_m divides a_n whenever m divides n —and this observation inspires an investigation of divisibility properties in more general integer sequences). “All good ideas in mathematics show up in a variety of mathematical and real-world contexts” (*Reflections*, p. 328).

Finally, a word of praise is due for the many illustrations appearing throughout in black and white, or in greyscale; these are carefully rendered and appropriate, never gratuitous. These books will be enjoyed by all, and in particular by the more visual and kinesthetic learners, through the exercises in hand-generating spirals, folding paper and sewing surfaces. Enjoy!