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**SYNOPSIS**

*C.-S. Lin*

If someone asks me how to verify the equality

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6},$$

immediately I would say it is easy by the Principle of Mathematical Induction, and that is true. If I am further asked how to get the equality in the first place, I would be probably hesitative for a while and I might fail to answer, unless I already knew a method. In fact, some strikingly original algebraic proofs are due to Archimedes and Fibonacci [D.M. Burton, *The History of Mathematics,* 2nd Edition, Wm.C. Brown Publishers, 1991, p. 104 and 102]. In numerical analysis we use factorial polynomials and the telescoping method [F. Scheid, *Numerical Analysis*, Schaum's Outline Series, McGraw-Hill Book Company, 1968, Chap. 17], and a graphical expression (not a proof, though) can be found in [R.B. Nelsen, *Proofs without Words*, Mathematical Association of America, 1993, p. 77]. Mathematical induction is a great common tool used for checking an equality like the one above. But it is imperfect in the sense that we have to know the equality in question beforehand. In this article, motivated by the method of proof of the equality above due to Chorlton [Math. Gazette, 71 (1987), 305–307, p. 305], we shall use finite sums of sine and cosine functions to produce several types of formulas involving summations of finite series of integers by way of differentiation. The next result is our basic tool.

Read on!



*Clifford Wagner*

The general public uses the word average for a concept that mathematicians prefer to call the arithmetic mean. Fortunately, mathematicians bring more than jargon to a discussion of averages. This note discusses two questions about arithmetic means. See how well you can answer them before reading the hints and answers.

Read on!



Featuring the problems of the 2001 Concours De Mathématiques Du Nouveau-Brunswick; and the problems of the 2001 Maritime Mathematics Contest.

Consider the following problem:

Let $a$, $b$, and $c$ be non-negative reals such that $a+b+c=1$. Show that $ab+ac+bc \leq 1/3$.

In this inequality problem, the variables satisfy a given constraint (their sum must be 1), and the expression in the inequality is a symmetric polynomial in those variables. In this article, we discuss a method of analyzing and proving certain inequalities of this type.

Read on!



This month's "free sample" is:

**2694**. *Proposed by Aaron Lee and Jason Wilson, students, Biola University, La Mirada, CA, USA.*

Given a line segment $AB$, construct a square $ABCD$ using four or fewer circular arcs and a straightedge. The construction should use fewer arcs than those usually given in texts.