

Engelhaupt remarked that in general, if  $n = 2^{a_1} + 2^{a_2} + \dots + 2^{a_r}$ , where  $0 \leq a_1 < a_2 < \dots < a_r$ , then  $n!$  finishes in  $n - r$  zeros since

$$\begin{aligned} E_2(n!) &= \sum_{t=1}^{\infty} \left\lfloor \frac{n}{2^t} \right\rfloor = \sum_{i=1}^r \sum_{t=1}^{\infty} \left\lfloor \frac{2^{a_i}}{2^t} \right\rfloor = \sum_{i=1}^r (2^{a_i-1} + 2^{a_i-2} + \dots + 1) \\ &= \sum_{i=1}^r (2^{a_i} - 1) = \left( \sum_{i=1}^r 2^{a_i} \right) - r = n - r. \end{aligned}$$

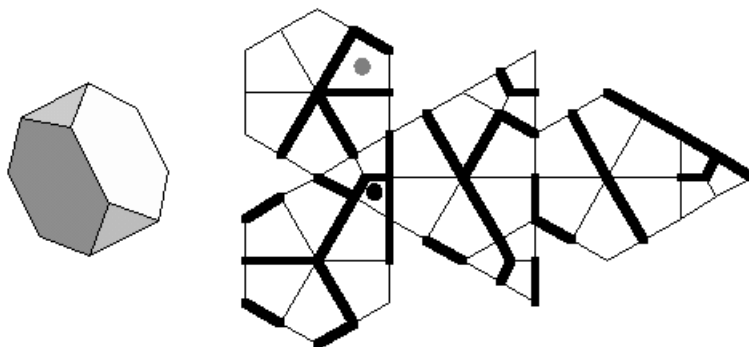
He gave the following example: Since  $n = 216 = 2^3 + 2^4 + 2^6 + 2^7$  has four summands,  $216!$  finishes in  $216 - 4 = 212$  zeros.

Reference:

- [1.] J. Robert, Elementary Number Theory — A Problem Oriented Approach, M.I.T. Press. Chapter X. Ex. 5, p. 76 and pp. 96–97.

## Another maze from Isador Hafner

How can you move from the dark spot to the light spot?



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