

THE SKOLIAD CORNER

No. 57

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Please include on any correspondence your name, school, grade, city, province or state and country. We are especially looking for solutions from high school students. Please send your solutions to the problems in this edition by *1 March 2002*. Look for prizes for solutions in the new year.

Our three entries this issue come from the BC mathematics competitions. My thanks go to Jim Totten of the University College of the Cariboo and Clint Lee of Okanagan University College for forwarding the material to me.

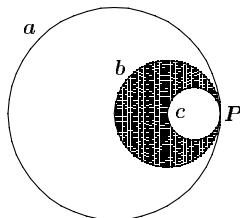
BRITISH COLUMBIA COLLEGES

Junior High School Mathematics Contest, 2001

Final Round – Part A

Friday May 4, 2001

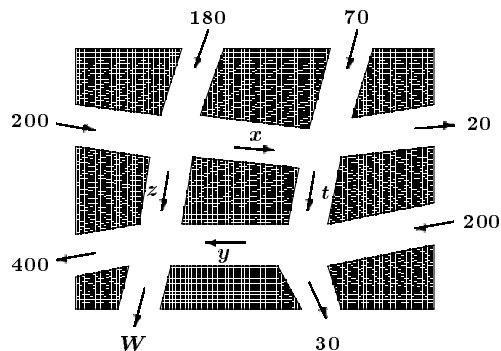
- The integer 9 is a perfect square that is both two greater than a prime number, 7, and two less than a prime number, 11. Another such perfect square is:
 (a) 25 (b) 49 (c) 81 (d) 121 (e) 169
- Three circles, a , b , and c , are tangent to each other at point P , as shown.



The center of b is on c and the center of a is on b . The ratio of the area of the shaded region to the total area of the unshaded regions enclosed by the circles is:

- (a) 3 : 13 (b) 1 : 3 (c) 1 : 4 (d) 2 : 9 (e) 1 : 25

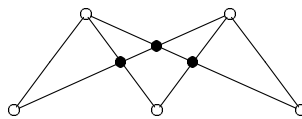
3. Here is a diagram of part of the downtown in a medium sized town in the interior of British Columbia. The arrows indicate one-way streets. The numbers or letters by the arrows represent the number of cars that travel along that portion of the street during a typical week day.



Assuming that no car stops or parks and that no cars were there at the beginning of the day, the value of the variable W is:

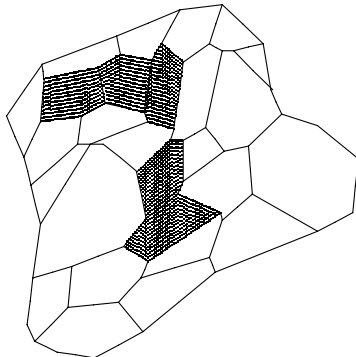
- (a) 30 (b) 200 (c) 250 (d) 350 (e) 600
4. The corners of a square of side x are cut off so that a regular octagon remains. The length of each side of the resulting octagon is:
- (a) $\frac{\sqrt{2}}{2}x$ (b) $2x(2 + \sqrt{2})$ (c) $\frac{x}{\sqrt{2} - 1}$
 (d) $x(\sqrt{2} - 1)$ (e) $x(\sqrt{2} + 1)$
5. The value of $(0.\overline{01})^{-1} + 1$ is: (The line over the digit 1 means that it is repeated indefinitely.)
- (a) $\frac{1}{91}$ (b) $\frac{90}{91}$ (c) $\frac{91}{90}$ (d) 10 (e) 91
6. The people living on Sesame Street all decide to buy new house numbers from the same store, and they purchase the digits for their house numbers in the order of their addresses: 1, 2, 3, If the store has 100 of each digit, then the first address which cannot be displayed occurs at house number:
- (a) 100 (b) 101 (c) 162 (d) 163 (e) 199

7. Given p dots on the top row and q dots on the bottom row, draw line segments connecting each top dot to each bottom dot. (In the diagram below, the dots referred to are the small open circles.) The dots must be arranged such that no three line segments intersect at a common point (except at the ends). The line segments connecting the dots intersect at several points. (In the diagram below, the points of intersection of the line segments are the small filled circles.) For example, when $p = 2$ and $q = 3$ there are three intersection points, as shown below.



When $p = 3$ and $q = 4$ the number of intersections is:

- (a) 7 (b) 12 (c) 18 (d) 21 (e) 27
8. At one time, the population of Petticoat Junction was a perfect square. Later, with an increase of 100, the population was 1 greater than a perfect square. Now, with an additional increase of 100, the population is again a perfect square. The original population was a multiple of:
- (a) 3 (b) 7 (c) 9 (d) 11 (e) 17
9. The cashier at a local movie house took in a total of \$100 from 100 people. If the rates were \$3 per adult, \$2 per teenager and 25 cents per child, then the smallest number of adults possible was:
- (a) 0 (b) 2 (c) 5 (d) 13 (e) 20
10. The island of Aresia has 27 states each of which belongs to one of two factions, the white faction and the grey faction, who are sworn enemies. The United Nations wishes to bring peace to Aresia by converting one state at a time to the opposite faction; that is, converting one state from white to grey or from grey to white, so that eventually all states belong to the same faction. In doing this they must guarantee that no single state is completely surrounded by states of the **opposite faction**. Note that a coastal state can never be completely surrounded, and that it may be necessary to convert a state from one faction to the other at one stage and then convert it back to its original faction later. A map of the state of Aresia is shown.



The five shaded states belong to the grey faction, and all of the unshaded states belong to the white faction. The minimum number of conversions necessary to completely pacify Aresia is:

- (a) 5 (b) 7 (c) 9 (d) 10 (e) 15

BRITISH COLUMBIA COLLEGES

Junior High School Mathematics Contest, 2001

Final Round – Part B

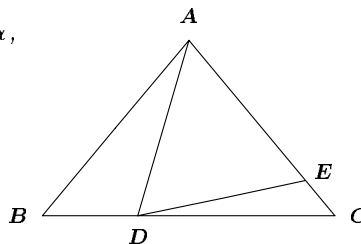
Friday May 4, 2001

- Find the smallest 3-digit integer which leaves a non-zero remainder when divided by any of 2, 3, 4, 5, or 6 but not when divided by 7.
- Assume that the land within two kilometres of the South Pole is flat. There are points in this region where you can travel one kilometre south, travel one kilometre east along *one* circuit of a latitude, and finally travel one kilometre north, and thus arrive at the point where you started. How far is such a point from the South Pole?
- Café de la Pêche offers three fruit bowls:
 - Bowl A has two apples and one banana;
 - Bowl B has four apples, two bananas, and three pears;
 - Bowl C has two apples, one banana, and three pears.

Your doctor tells you to eat exactly 16 apples, 8 bananas and 6 pears each day. How many of each type of bowl should you buy so there is no fruit left over? Find all possible answers. (The numbers of bowls must be non-negative integers.)

4. In the triangle shown, $\angle BAD = \alpha$,
 $\overline{AB} = \overline{AC}$ and $\overline{AD} = \overline{AE}$.

Find $\angle CDE$ in terms of α .



5. In the multiplication below each of the letters stands for a distinct digit.
 Find all values of $JEEP$.

$$\begin{array}{r} JEEP \\ \times JEEP \\ \hline BEEBEEP \end{array}$$

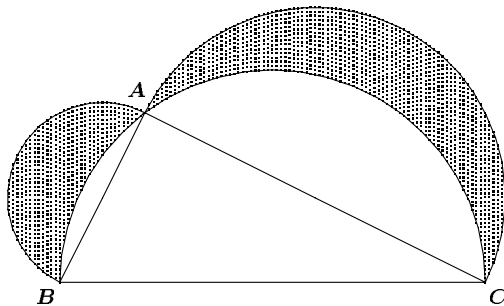
BRITISH COLUMBIA COLLEGES

Senior High School Mathematics Contest, 2001

Final Round – Part B

Friday May 4, 2001

- See question #4 above.
- A semicircle BAC is mounted on the side BC of the triangle ABC . Semicircles are also mounted outwardly on the sides BA and AC , as shown in the diagram. The shaded crescents represent the area inside the smaller semicircles and outside the semicircle BAC . Show that the total shaded area equals the area of the triangle ABC .



3. Five schools competed in the finals of the British Columbia High School Track Meet. They were Cranbrook, Duchess Park, Nanaimo, Okanagan Mission, and Selkirk. The five events in the finals were: the high jump, shot put, 100-metre dash, pole vault and 4-by-100 relay. In each event the school placing first received five points; the one placing second, four points; the one placing third, three points; and so on. Thus, the one placing last received one point. At the end of the competition, the points of each school were totalled, and the totals determined the final ranking.
- (a) Cranbrook won with a total of 24 points.
 - (b) Sally Sedgwick of Selkirk won the high jump hands down (and feet up), while Sven Sorenson, also of Selkirk, came in third in the pole vault.
 - (c) Nanaimo had the same number of points in at least four of the five events.
- Each school had exactly one entry in each event. Assuming there were no ties and the schools ended up being ranked in the same order as the alphabetical order of their names, in what position did Doug Dolan of Duchess Park rank in the high jump?
4. A box contains tickets of two different colours: blue and green. There are 3 blue tickets. If two tickets are to be drawn together at random from the box, the probability that there is one ticket of each colour is exactly $\frac{1}{2}$. How many green tickets are in the box? Give all possible solutions.
5. In (a), (b), and (c) below the symbols m , h , t , and u can represent any integer from 0 to 9 inclusive.
- (a) If $h - t + u$ is divisible by 11, prove that $100h + 10t + u$ is divisible by 11.
 - (b) If $h + u = m + t$, prove that $1000m + 100h + 10t + u$ is divisible by 11.
 - (c) Is it possible for $1000m + 100h + 10t + u$ to be divisible by 11 if $h + u \neq m + t$? Explain.