27: No 5 SEPTEMBER / SEPTEMBRE 2001

Published by:

Canadian Mathematical Society
Société mathématique du Canada
577 King Edward, POB/CP 450-A
Ottawa, ON K1N 6N5
Fax/Télec: 613 565 1539

SYNOPSIS

In the textbook [*Mathematics for Elementary Teachers*, second edition, Saunders College Publishing, (1997) by Thomas Sonnabend], one can find the following problem:

Let A, B, C, a, b, c, α, β, γ be distinct digits from 1, 2, 3, 4, 5, 6, 7, 8, 9. Give an example such that the equality below holds.

$$\begin{array}{r} A\,a\,\alpha \\ +\,B\,b\,\beta \\ \hline C\,c\,\gamma \end{array}$$

Students turned in many different results. Some of them gave even 20 solutions. The record in our class of thirty students is made by the second author of this article, who found 120 such examples.

Thus, the question is: How many such examples are there?

Read on!

315 The Skoliad Corner: No. 55 *Shawn Godin*

Featuring the porblems of the 2000 National Bank Junior Mathematics Competition from New Zealand; and the problems of part A of the final round of the BC senior mathematics competition.

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326 An Extension of Ptolemy's Theorem *David Loeffler*

Ptolemy's Theorem is a well known result that states that if $ABCD$ is a convex cyclic quadrilateral (with vertices in this order), then $AC \cdot BD = AB \cdot CD + AD \cdot BC$.

This may be generalized in the following way: let $ABCD$ be any quadrilateral, not necessarily cyclic, not even necessarily convex. Let $A = \angle BAD$ and $C = \angle DCB$. (Note: If $ABCD$ is not convex, care should be taken that angles are measured in the right sense; that is, so that $\angle BAD$ and $\angle DCB$ have the same orientation.) Then we have

$$AC^2 \cdot BD^2 = AB^2 \cdot CD^2 + AD^2 \cdot BC^2 - 2AB \cdot BC \cdot CD \cdot DA \cos(A+C) .$$

This immediately implies Ptolemy's Theorem, since if $ABCD$ is cyclic, $A + C = \pi$, so that $\cos(A + C) = -1$.

I do not know if this formula is known; I certainly have not seen it before, and I have not met anybody who has.

Ed. – See Crux problem 1015 [1985 : 128]: "the following extension of Ptolemy's Theorem has recently appeared (E. Kraemer, Zobecneni Vety Ptolemaivoy, *Rozhledy Mathematickofyzikalni* (Czechoslovakia) 63, no. 8, 345-349):

$$a_{13}^2 a_{24}^2 = a_{12}^2 a_{34}^2 + a_{23}^2 a_{41}^2 - 2a_{12}a_{23}a_{34}a_{41} \cos(A_1 + A_3) . "$$

For the proof, read on!

328 Convergent and Divergent Infinite Series
Sandra Pulver

This is a gentle introduction to the theory of infinite series.

This month's "free sample" is:

2651★. *Proposed by Murray S. Klamkin, University of Alberta, Edmonton, Alberta.* Dedicated to Professor M.V. Subbarao on the occasion of his 80th birthday. (Professor Klamkin offers a prize of $100 for the first correct solution received by the Editor-in-Chief.)

Let P be a non-exterior point of a regular n–dimensional simplex $A_0A_1A_2\ldots A_n$ of edge length e. If

$$F \;=\; \sum_{k=0}^{n} PA_k + \min_{0\le k\le n} PA_k\,, \qquad F' \;=\; \sum_{k=0}^{n} PA_k + \max_{0\le k\le n} PA_k\,,$$

determine the maximum and minimum values of F and F'.

This problem was suggested by problem 2594 for a general triangle, and the proposer was trying to obtain a stronger inequality by finding the maximum of F.