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SYNOPSIS

289 The Academy Corner: No. 42 *Bruce Shawyer*

Featuring the problems of the Undergraduate Mathematics Competition held at Memorial University of Newfoundland on 27 March 2001; and some more solutions to problems of the 2000 Atlantic Provinces Council on the Sciences Mathematics Competition.

292 The Olympiad Corner: No. 215 *R.E. Woodrow*

Featuring a third set of Klamkin Quickies; the problems of the Swedish Mathematical Competition, Final Round 1997; selected problems of the Ukrainian Mathematical Olympiad 1998; the problems of the Vietnamese Mathematical Olympiad, Category A, 1998; the problems of Category B of the Vietnamese Mathematical Olympiad 1998; Klamkin's answers for the third set of Quickies; the answers to the first five Klamkin Quickies given in the April issue; a comment answering an editorial question we posed; readers' solutions to the St. Petersburg City Mathematical Olympiad, Third Round; and solutions to problems of the Selective Round, 11th Grade of the St. Petersburg City Mathematical Olympiad.

309 Book Reviews *Alan Law*

Geometry at Work by Catherine A. Gorini

311 An Interesting Arithmetic Problem

by *Jingcheng Tong, Edith Atkins and Debora Simonson*

In the textbook [*Mathematics for Elementary Teachers*, second edition, Saunders College Publishing, (1997) by Thomas Sonnabend], one can find the following problem:

Let $A, B, C, a, b, c, \alpha, \beta, \gamma$ be distinct digits from 1, 2, 3, 4, 5, 6, 7, 8, 9. Give an example such that the equality below holds.

$$\begin{array}{r} A a \alpha \\ + B b \beta \\ \hline C c \gamma \end{array}$$

Students turned in many different results. Some of them gave even 20 solutions. The record in our class of thirty students is

made by the second author of this article, who found 120 such examples.

Thus, the question is: How many such examples are there?

Read on!

315 The Skoliad Corner: No. 55 *Shawn Godin*

Featuring the problems of the 2000 National Bank Junior Mathematics Competition from New Zealand; and the problems of part A of the final round of the BC senior mathematics competition.

312 Mathematical Mayhem

321 Mayhem Problems

324 Problem of the Month *Jimmy Chui*

325 Writer's Guide For Mayhem *Shawn Godin*

326 An Extension of Ptolemy's Theorem *David Loeffler*

Ptolemy's Theorem is a well known result that states that if $ABCD$ is a convex cyclic quadrilateral (with vertices in this order), then $AC \cdot BD = AB \cdot CD + AD \cdot BC$.

This may be generalized in the following way: let $ABCD$ be any quadrilateral, not necessarily cyclic, not even necessarily convex. Let $A = \angle BAD$ and $C = \angle DCB$. (Note: If $ABCD$ is not convex, care should be taken that angles are measured in the right sense; that is, so that $\angle BAD$ and $\angle DCB$ have the same orientation.) Then we have

$$AC^2 \cdot BD^2 = AB^2 \cdot CD^2 + AD^2 \cdot BC^2 - 2AB \cdot BC \cdot CD \cdot DA \cos(A+C).$$

This immediately implies Ptolemy's Theorem, since if $ABCD$ is cyclic, $A + C = \pi$, so that $\cos(A + C) = -1$.

I do not know if this formula is known; I certainly have not seen it before, and I have not met anybody who has.

Ed. – See Crux problem 1015 [1985 : 128]: “the following extension of Ptolemy's Theorem has recently appeared (E. Kraemer, *Zobecní Věty Ptolemaiovoy, Rozhledy Mathematickofyzikalni* (Czechoslovakia) 63, no. 8, 345-349):

$$a_{13}^2 a_{24}^2 = a_{12}^2 a_{34}^2 + a_{23}^2 a_{41}^2 - 2a_{12} a_{23} a_{34} a_{41} \cos(A_1 + A_3). ”$$

For the proof, read on!

328 Convergent and Divergent Infinite Series

Sandra Pulver

This is a gentle introduction to the theory of infinite series.

335 Problems: 2651–2663

This month's "free sample" is:

2651★. *Proposed by Murray S. Klamkin, University of Alberta, Edmonton, Alberta.* Dedicated to Professor M.V. Subbarao on the occasion of his 80th birthday. (Professor Klamkin offers a prize of \$100 for the first correct solution received by the Editor-in-Chief.)

Let P be a non-exterior point of a regular n -dimensional simplex $A_0A_1A_2 \dots A_n$ of edge length e . If

$$F = \sum_{k=0}^n PA_k + \min_{0 \leq k \leq n} PA_k, \quad F' = \sum_{k=0}^n PA_k + \max_{0 \leq k \leq n} PA_k,$$

determine the maximum and minimum values of F and F' .

This problem was suggested by problem 2594 for a general triangle, and the proposer was trying to obtain a stronger inequality by finding the maximum of F .

338 Solutions: 2542–2554