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SYNOPSIS

161 The Academy Corner: No. 40 *Bruce Shawyer*

Featuring some readers' solutions by Edward T.H. Wang, Wilfrid Laurier University, Waterloo, Ontario to problems of the Memorial University Undergraduate Mathematics Competition, written in March 2000; and readers' solutions to problems of the Memorial University Undergraduate Mathematics Competition, written in September 2000.

166 The Olympiad Corner: No. 213 *R.E. Woodrow*

Featuring a set of five Klamkin Quickies; the problems of the two days of the Vietnamese Mathematical Olympiad 1997; the problems of the Team Selection Examination for Turkey for the 38th IMO; the problems of the Chilean Mathematical Olympiads 1994–1995; readers' solutions to problems of the Third Macedonian Mathematical Olympiad; and readers' solutions to problems of the Ninth Irish Mathematical Olympiad.

184 Book Reviews *Alan Law*

Revolutions in Differential Equations: Exploring ODEs with Modern Technology edited by Michael J. Kallaher

188 On Improper Integrals

by Javad Mashreghi

In this note, we evaluate some improper integrals. For the first proposition, two proofs are given. The first one is a nice application of **integration by parts** and **change of variable** techniques. The second method is more advanced. We make use of the **Mean Value Theorem** and the **Zero Derivative Theorem** and **Chain Rule**. It is shown that $I(a)$ is a differentiable function, and to find its derivative one simply changes the order of integration and differentiation. An application of the **First Fundamental Theorem of Calculus** leads to more interesting results.

Proposition 1 Let a be a positive real number. Then we have that

$$I(a) = \int_0^{\infty} \frac{\arctan\left(\frac{x}{a}\right) + \arctan(ax)}{1+x^2} dx = \frac{\pi^2}{4}.$$

Read on!

Featuring the “official solutions” to the problems of the Final Round of the Senior Mathematics Contest of the British Columbia Colleges.

199 Mathematical Mayhem

199 Problem of the Month *Jimmy Chui*

201 Mayhem Problems

201 High School Problems

H285–H288

202 Advanced Problems

A261–A264

203 Challenge Board Problems

The Konhauser Problemfest

207 The Volume of a Tetrahedron and Areas of its Faces

C.-S. Lin

A classical result states that if a tetrahedron $ABCD$ is rectangular at the vertex B , so that all the edges meeting at B are mutually perpendicular (See Figure 1), then the relation $b^2 = a^2 + c^2 + d^2$ is called the S -dimensional version of Pythagorean Theorem, where x denotes the area of the face opposite to the vertex X of the tetrahedron $ABCD$. The classical Heron’s Formula of plane geometry says that for a triangle having sides of length a , b and c , and area K , we have

$$K = \frac{1}{4} (2(a^2b^2 + b^2c^2 + c^2a^2) - (a^4 + b^4 + c^4))^{\frac{1}{2}}, \text{ or}$$

$$K = (s(s - a)(s - b)(s - c))^{\frac{1}{2}} \text{ for } s = \frac{1}{2}(a + b + c).$$

Although a well known classical result indicates that the volume of a tetrahedron can be expressed in terms of its six edges, I have presented elsewhere a relation which I called the S -dimensional analogue of Heron’s Formula. Indeed, if V denotes the volume of an H -tetrahedron (attributed to Heron) $ABCD$; that is, it can be formed by gluing together two tetrahedra $ACDE$ and $BCDE$ both rectangular at the vertex E , and on a common congruent face $\triangle CDE$ (see figure 2) so that $\triangle ABC \perp \triangle ABD$, then

$$(*) V^2 = \frac{2cd (4(s - a)(s - b)(s - c)(s - d) - c^2d^2 - 2abcd)^{\frac{1}{2}}}{9(c^2 + d^2)^{\frac{1}{2}}}$$

for $s = (a + b + c + d)$, and the small letters denote areas of faces as described before.

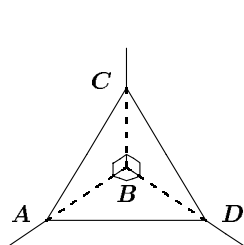


Figure 1

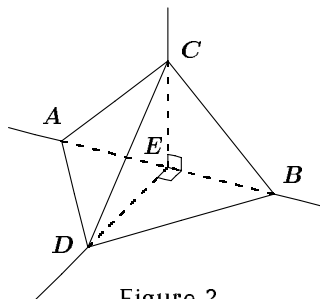


Figure 2

The identity (*) leads naturally to the following question: Can we establish a formula which expresses the volume of an arbitrary tetrahedron in terms of its four faces? In this article I shall construct two formulas for an L -tetrahedron (See definition below). Each one shows the volume of an L -tetrahedron in terms of areas of its faces and some constant.

48 Problems: 2626—2636

This month's "free sample" is:

2631 Proposed by Edward T.H. Wang, Wilfrid Laurier University, Waterloo, Ontario.

Find the exact value of $\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{k} \binom{n-k}{k} 2^{n-2k}$.

51 Solutions: 2513, 2520, 2528–2530, 2532, 2536