BOOK REVIEWS

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Revolutions in Differential Equations:
Exploring ODEs with Modern Technology,
edited by Michael J. Kallaher,
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ISBN 0-88385-160-1 softcover, 100 pages, $18.75 (US).
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It is generally acknowledged that numerical techniques should comprise an essential part of the undergraduate education for all students in mathematics and the physical sciences. The importance of the skill sets required to perform scientific computation has grown over the last decade, with the ubiquitous presence of computers in both industry and academia. The raw computer power that a modest desktop, or even laptop, provides would have been hard to credit a generation ago.

Prior to the revolution in microcomputer technology, the vast majority of numerical problems in the mathematical and physical sciences were exclusively the domain of large mainframe computers. To apply the techniques of numerical analysis required familiarity with programming languages such as Fortran or C, combined with a knowledge and understanding of a wide variety of sophisticated numerical techniques. Different indeed from the current situation, where programs such as Mathematica, Maple and Matlab, provide robust root finding algorithms, numerical integration packages, ode and pde solvers, numerical evaluation of special functions and a host of matrix manipulation routines, all tightly integrated with sophisticated graphing and visualisation procedures and, in the case of Mathematica and Maple, an advanced computer algebra system. All of this running on machines that can be purchased for $1,000 and up!

While it is certainly the case that these packages are not without their pitfalls for the unwary, they nevertheless redefine the resources that a typical undergraduate student can bring to bear on problems in applied mathematics and the physical sciences. While this revolution in technology holds out the promise to transform the way mathematics is taught and utilised within the undergraduate science curriculum, there exists a reluctance on the part of many educators to fully exploit the potential that it offers.

The reasons behind this reluctance are many and complex. In part they arise from a well justified uneasiness with providing students, often with limited mathematical skills, a black box to solve complex mathematical problems. This is compounded by the fact there does not exist a clear consensus as to how these techniques are best integrated into the undergraduate curriculum at a fundamental level.

The text "Revolutions in Differential Equations: Exploring ODEs with Modern Technology" is a short collection of papers in the MAA Notes series. The series is designed to address issues in undergraduate mathematics. The text consists of eight articles by a number of authors describing their experience in the development and application of numerical techniques and visualisation in the teaching of ODEs in the undergraduate mathematics curriculum. The authors are all active researchers in the area of ODEs and have had considerable experience in integrating modern analytical, numerical and graphical methods into the teaching of ODEs. The papers are replete with examples as well as informed discussion of the approaches that this technology allows. As such, the text serves as an excellent introduction to instructors wishing to incorporate computer based methods into their teaching.

Four of the eight papers (Borrelli and Coleman, Boyce, Branton and Hale, Manoranjan) describe in considerable detail how ODE solvers and graphical tools can be used to analyse and visualise the solutions of a wide range of very diverse ODEs, in the context of the undergraduate mathematics curriculum. A common element in all these papers is how this technology allows students to study a range of complex topics in ODEs such as bifurcation, stability, oscillations, fixed points, etc., in a relatively informal manner. The approaches described by the different authors encourage, in many instances require, students to explore the solutions to select ODEs for different initial conditions and parameters, and in this way develop insight into the nature of the solutions to ODEs. Much of this material would be inaccessible to undergraduates within the framework of a conventional ODE course, given the limited range of analytic techniques that can realistically be covered. In addition, the techniques described in these articles provide students with a powerful set of tools with which to tackle a wide variety of problems in mathematical modeling and scientific computation.

The article by Lomen examines another aspect in the use of computers in the study of ODEs at the undergraduate level. In this article Lomen describes how computers facilitate the use of data in constructing mathematical models that can be expressed in terms of ODEs. The data serves not only to motivate the model but as a check on the applicability of the model. This approach is echoed in the article by Cooper and LoFaro, who provide an example of how the Web can serve as a useful source of data and problems in mathematical modeling. As with the four papers referred to previously, the examples chosen demonstrate how the use of computer based ODE solvers and visualistion software allow students to explore and analyse "real world" problems mathematically.

A reminder of the many complexities in numerical analysis is provided by the article by Shampine and Gladwell, who discuss general aspects regarding the teaching of numerical methods. The article contains a brief overview of the fundamentals of numerical analysis and touches on some of the difficulties inherent in the various approaches to the numerical solution of ODEs. The article also includes a description of some quality software packages that are available to solve ODEs and how to locate these packages. The authors also touch on the ODE solvers used in Mathematica and Maple and how such programs can utilise the more sophisticated routines included in the IMSL and NAG libraries.

All in all, I found this an extremely interesting and informative book. The articles are, on the whole, well written and provide explicit examples, as well as many useful and practical ideas from several innovative and capable practitioners in this rapidly evolving area of undergraduate education.

My only disappointment with the book is that it does not discuss the effect of this technology on student outcomes. While the article by West, "Technology in Differential Equation Courses: My experiences, student reactions" includes some discussion of the student experience, it is largely anecdotal in nature. While I do not doubt the effectiveness of the approaches described in the various articles, I would like to see some conclusive data that clearly demonstrates the effectiveness of these techniques. Is it a demonstratively better way of teaching ODEs than the traditional analytical approach? Do these methods improve students' attitudes towards mathematics? Does the use of this technology provide students with a useful skill set that they can apply to problems in mathematical modeling and scientific computation? I would like to think that the answer to these questions is a definite yes. Possibly it is.

Who wrote this?

The biologist can push it back to the original protist, and the chemist can push it back to the crystal, but none of them touch the real question of why or how the thing began at all. The astronomer goes back untold million of years and ends in gas and emptiness, and then the mathematician sweeps the whole cosmos into unreality and leaves one with mind as the only thing of which we have any immediate apprehension. *Cogito ergo sum, ergo omnia esse videntur*. All this bother, and we are no further than Descartes. Have you noticed that the astronomers and mathematicians are much the most cheerful people of the lot? I suppose that perpetually contemplating things on so vast a scale makes them feel either that it doesn't matter a hoot anyway, or that anything so large and elaborate must have some sense in it somewhere.