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SYNOPSIS

In 1877 Dobiñski stated that there exist integers q_n such that

$$\frac{0^n}{0!}+\frac{1^n}{1!}+\frac{2^n}{2!}+\frac{3^n}{3!}+\cdots \;=\; q_n\left(\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots\right) \;=\; q_n e\,,$$

and he calculated their values for $n = 1$ through 5. Indeed, q_n are the Bell numbers, so named in honour of the American mathematician Eric Temple Bell (1883-1960), who was among the first to popularize these numbers. It may be shown that q_n is just the sum of Stirling numbers of the second kind:

$$q_n \;=\; \left\{{n \atop 1}\right\}+\left\{{n \atop 2}\right\}+\cdots+\left\{{n \atop n}\right\}.$$

Since $\left\{{n \atop k}\right\}$ is the number of k–member quotient sets of an n–set, the Bell number q_n is the number of all quotient sets of an n–set. It can also be calculated via recursion in terms of the Stirling numbers of the first kind; the following formula is due to G.T. Williams:

$$\begin{bmatrix} n \\ n \end{bmatrix} q_n - \begin{bmatrix} n \\ n-1 \end{bmatrix} q_{n-1} + - \cdots + (-1)^{n-1} \begin{bmatrix} n \\ 1 \end{bmatrix} q_1 \;=\; 1 \,,$$

where $\left[{n \atop k}\right]$ are the coefficients of the polynomial of degree n with roots $0\,,\,1\,,\,2\,,\,\ldots\,,\,(n-1)$:

$$\begin{bmatrix} n \\ n \end{bmatrix} x^n - \begin{bmatrix} n \\ n-1 \end{bmatrix} x^{n-1} + - \cdots + (-1)^{n-1} \begin{bmatrix} n \\ 1 \end{bmatrix} x \;=\; x(x-1)(x-2)\cdots(x-n+1)\,.$$

For more, read the article!

In this article, we introduce two important classes of combinatorial numbers which appear frequently: derangements and Stirling numbers. We also hope to emphasize the importance and usefulness of basic counting principles.

We can think of a permutation on n objects as a 1-1 function π from the set $\{1, 2, \ldots, n\}$ to itself. For example, for $n = 3$, the map π given by $\pi(1) = 1$, $\pi(2) = 3$, and $\pi(3) = 2$ is a permutation on 3 objects, namely the elements of $\{1, 2, 3\}$; a permutation essentially re-arranges the elements. Note that there are $n!$ different permutations on n objects. Then, a *derangement* is a permutation π which has no fixed points; that is, $\pi(i) \neq i$ for all $i = 1, 2, \ldots, n$. Alternatively, if we think of a permutation on n objects as a distribution of n letters to n corresponding envelopes, then a derangement is a permutation where no letter is inserted into the correct corresponding envelope. Let D_n denote the number of derangements on n objects. The natural question to ask is, what is the formula for D_n?

This month's "free sample" is:

2455. *Proposed by Gerry Leversha, St. Paul's School, London, England.*

Three equal circles, centred at A, B and C intersect at a common point P. The other intersection points are L (not on circle centre A), M (not on circle centre B), and N (not on circle centre C). Suppose that Q is the centroid of $\triangle LMN$, that R is the centroid of $\triangle ABC$, and that S is the circumcentre of $\triangle LMN$.

(a) Show that P, Q, R and S are collinear.

(b) Establish how they are distributed on the line.