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SYNOPSIS

257 The Academy Corner: No. 26 *Bruce Shawyer*

Abstract of the talks at the 1998 Canadian Undergraduate Mathematics Conference, held at the University of British Columbia in July 1998. (Part 4)

261 The Olympiad Corner: No. 199 *R.E. Woodrow*

Featuring the Danish Georg Mohr Konkurrencen Mathematik 1996; the St. Petersburg City Mathematical Olympiad, third round, 1996, and the selective round, 1996; readers' solutions to the 1994 Balkan Olympiad; and problems from the Fourth Grade of the 38th Mathematics Competition of the Republic of Slovenia.

270 Book Review *Alan Law*

In Polya's Footsteps, by *Ross Honsberger*

Reviewed by *Murray Klamkin*, University of Alberta.

274 *Apropos* Bell and Stirling Numbers

by *Antal E. Fekete*

In 1877 Dobiński stated that there exist integers q_n such that

$$\frac{0^n}{0!} + \frac{1^n}{1!} + \frac{2^n}{2!} + \frac{3^n}{3!} + \cdots = q_n \left(\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots \right) = q_n e,$$

and he calculated their values for $n = 1$ through 5. Indeed, q_n are the Bell numbers, so named in honour of the American mathematician Eric Temple Bell (1883-1960), who was among the first to popularize these numbers. It may be shown that q_n is just the sum of Stirling numbers of the second kind:

$$q_n = \left\{ \begin{matrix} n \\ 1 \end{matrix} \right\} + \left\{ \begin{matrix} n \\ 2 \end{matrix} \right\} + \cdots + \left\{ \begin{matrix} n \\ n \end{matrix} \right\}.$$

Since $\left\{ \begin{matrix} n \\ k \end{matrix} \right\}$ is the number of k -member quotient sets of an n -set, the Bell number q_n is the number of all quotient sets of an n -set. It can

also be calculated via recursion in terms of the Stirling numbers of the first kind; the following formula is due to G.T. Williams:

$$\begin{bmatrix} n \\ n \end{bmatrix} q_n - \begin{bmatrix} n \\ n-1 \end{bmatrix} q_{n-1} + \dots + (-1)^{n-1} \begin{bmatrix} n \\ 1 \end{bmatrix} q_1 = 1,$$

where $\begin{bmatrix} n \\ k \end{bmatrix}$ are the coefficients of the polynomial of degree n with roots $0, 1, 2, \dots, (n-1)$:

$$\begin{bmatrix} n \\ n \end{bmatrix} x^n - \begin{bmatrix} n \\ n-1 \end{bmatrix} x^{n-1} + \dots + (-1)^{n-1} \begin{bmatrix} n \\ 1 \end{bmatrix} x = x(x-1)(x-2)\dots(x-n+1).$$

For more, read the article!

282 The Skoliad Corner: No. 39 *R.E. Woodrow*

288 Announcement — ATOM Volume II

Algebra — Intermediate Methods

by *Bruce Shawyer*

289 Mathematical Mayhem

289 Shreds and Slices

Another Combinatorial Proof

291 Mayhem Problems

291 High School Problems **H253–H256**

292 Advanced Problems **A229–A232**

293 Challenge Board Problems **C85–C86**

lems

294 Problem of the Month *Jimmy Chui*

295 J.I.R. McKnight Problems Contest 1986 — Solutions

297 J.I.R. McKnight Problems Contest 1989

299 Derangements and Stirling Numbers *Naoki Sato*

In this article, we introduce two important classes of combinatorial numbers which appear frequently: derangements and Stirling numbers. We also hope to emphasize the importance and usefulness of basic counting principles.

We can think of a permutation on n objects as a 1-1 function π from the set $\{1, 2, \dots, n\}$ to itself. For example, for $n = 3$, the map π given by $\pi(1) = 1$, $\pi(2) = 3$, and $\pi(3) = 2$ is a permutation on 3 objects, namely the elements of $\{1, 2, 3\}$; a permutation essentially re-arranges the elements. Note that there are $n!$ different permutations on n objects. Then, a *derangement* is a permutation π which has no fixed points; that is, $\pi(i) \neq i$ for all $i = 1, 2, \dots, n$. Alternatively, if we think of a permutation on n objects as a distribution of n letters to n corresponding envelopes, then a derangement is a permutation where no letter is inserted into the correct corresponding envelope. Let D_n denote the number of derangements on n objects. The natural question to ask is, what is the formula for D_n ?

306 Problems: 2446, 2451–2462

This month's "free sample" is:

2455. *Proposed by Gerry Leversha, St. Paul's School, London, England.*

Three equal circles, centred at A , B and C intersect at a common point P . The other intersection points are L (not on circle centre A), M (not on circle centre B), and N (not on circle centre C). Suppose that Q is the centroid of $\triangle LMN$, that R is the centroid of $\triangle ABC$, and that S is the circumcentre of $\triangle LMN$.

(a) Show that P , Q , R and S are collinear.

(b) Establish how they are distributed on the line.

309 Solutions: 2324, 2339, 2346, 2348, 2351–55