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SYNOPSIS

129 The Academy Corner: No. 24 *Bruce Shawyer*

Abstract of the talks at the 1998 Canadian Undergraduate Mathematics Conference, held at the University of British Columbia in July 1998. (Part 2)

132 The Olympiad Corner: No. 197 *R.E. Woodrow*

Featuring the 47th Polish Mathematical Olympiad, 1996; the 10th Nordic Mathematical Contest, 1996; the Dutch Mathematical Olympiad, 1995; some readers' solutions to problems proposed to the jury, but not used, at the 1996 IMO in Mumbai, India; a reader's solution to a problem of the 17th Austiran-Polish Mathematics Competition; readers' solutions to problems of the Iranian Mathematical Olympiad, 1994; and readers' solutions to probelms of the Japan Mathematical Olympiad, 1994.

146 Book Review *Alan Law*

Interdisciplinary Lively Application Projects, edited by *David C. Amey*

Reviewed by *T.W. Leung*, Hong Kong Polytechnic University.

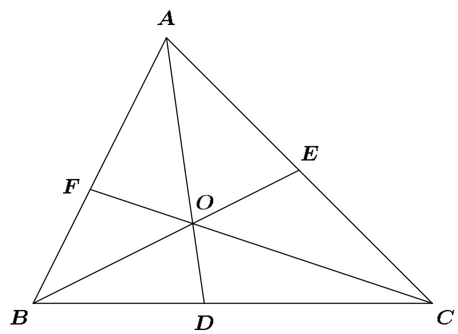
148 Euler's Triangle Theorem

by *G.C. Shephard*

In 1780, Leonhard Euler proved the following remarkable result:

Theorem: Let $[A, B, C]$ be an arbitrary triangle and O any point of the plane which does not lie on a side of the triangle. Let AO, BO, CO meet BC, CA and AB in the points D, E, F respectively. Then

$$\frac{||AO||}{||OD||} + \frac{||BO||}{||OE||} + \frac{||CO||}{||OF||} = \frac{||AO||}{||OD||} \cdot \frac{||BO||}{||OE||} \cdot \frac{||CO||}{||OF||} + 2 . \qquad (1)$$



Euler's proof of his theorem is by algebra and trigonometry; he calculates the ratios of the lengths of the line segments in (1) using trigonometrical formulae involving the sines of the angles between the lines at O. The proof takes two and a half pages, and whilst we do not wish to criticize the work of one of the most illustrious of mathematicians, it is worth considering the shortcomings of his proof. Apart from its length and complexity, the proof gives no insight as to *why* relation (1) is true, and therefore does nothing to suggest the existence of other relations of a similar nature. The approach given seems to overcome these criticisms; it depends on what has been called the "area principle" and the "area method".

Featuring the Alberta High School Mathematics Competition 1998; the last three problems of the Olympiade Mathematique Belge, omitted from the last issue; and the answers to that competition.

There are four famous triangular centres that all students of classical geometry should be familiar with; the centroid, the circumcentre, the incentre, and the orthocentre.

Although it was taken for granted in the definitions, it is not trivial to show that the points above exist. After all, why do three line segments have to intersect at a common point? We say the lines concur if they do. In fact, draw three lines from the vertices of a triangle to their opposite sides. These lines are called cevians. It is easy to see that these cevians need not concur.

There are many ways to show that the points described above exist and to show that certain cevians always concur for any triangle. Here we will consider something of purely mathematical interest. There is a certain similarity between the definitions of the four centres and so it may be possible that the existence of one centre is enough to prove the existence of another. For example, if it has been shown that the centroid exists, then is it possible to show that the incentre exists from this fact? We present two such scenarios here.

This month's "free sample" is:

2432. *Proposed by K.R.S. Sastry, Bangalore, India.*

In $\triangle ABC$, we use the standard notation: O is the circumcentre, H is the orthocentre. Let M be the mid-point of BC, $OH = m$, $OM = n$ ($m, n \in \mathbb{N}$), and suppose that $OH \parallel BC$.

How many sides of $\triangle ABC$ can have integer lengths?