

THE SKOLIAD CORNER

No. 37

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We begin this number with the problems of Part I of the Alberta High School Prize Exam, written November 1998. My thanks go to the organizing committee, chaired by Ted Lewis, University of Alberta, Edmonton, for supplying the contest and its problems.

THE ALBERTA HIGH SCHOOL MATHEMATICS COMPETITION Part I — November 1998

1. A restaurant usually sells its bottles of wine for 100% more than it pays for them. Recently it managed to buy some bottles of its most popular wine for half of what it usually pays for them, but still charged its customers what it would normally charge. For these bottles of wine, the selling price was what percent more than the purchase price?

(a) 50 (b) 200 (c) 300 (d) 400 (e) not enough information

2. How many integer solutions n are there to the inequality $34n \geq n^2 + 289$?

(a) 0 (b) 1 (c) 2 (d) 3 (e) more than 3

3. A university evaluates five magazines. Last year, the rankings were MacLuck with a rating of 150, followed by MacLock with 120, MacLick with 100, MacLeck with 90, and MacLack with 80. This year, the ratings of these magazines are down 50%, 40%, 20%, 10% and 5% respectively. How does the ranking change for MacLeck?

(a) up 3 places (b) up 2 places (c) up 1 place (d) unchanged (e) down 1 place

4. Parallel lines are drawn on a rectangular piece of paper. The paper is then cut along each of the lines, forming n identical rectangular strips. If the strips have the same length to width ratio as the original, what is this ratio?

(a) $\sqrt{n} : 1$ (b) $n : 1$ (c) $n : \frac{\sqrt{5+1}}{2}$ (d) $n : 2$ (e) $n^2 : 1$

5. "The smallest integer which is at least $a\%$ of 20 is 10." For how many integers a is this statement true?

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

6. Let $S = 1 + 2 + 3 + \cdots + 10^n$. How many factors of 2 appear in the prime factorization of S ?

- (a) 0 (b) 1 (c) $n - 1$ (d) n (e) $n + 1$

7. When $1 + x + x^2 + x^3 + x^4 + x^5$ is factored as far as possible into polynomials with integral coefficients, what is the number of such factors, not counting trivial factors consisting of the constant polynomial 1?

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

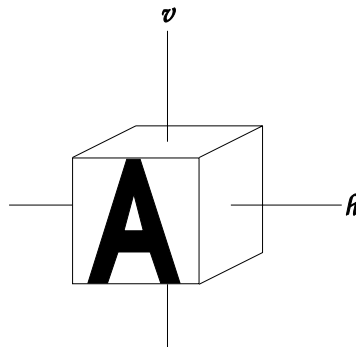
8. In triangle ABC , $AB = AC$. The perpendicular bisector of AB passes through the midpoint of BC . If the length of AC is $10\sqrt{2}$ cm, what is the area of ABC in cm^2 ?

- (a) 25 (b) $25\sqrt{2}$ (c) 50 (d) $50\sqrt{2}$ (e) none of these

9. If $f(x) = x^x$, what is $f(f(x))$ equal to?

- (a) x^{2x} (b) $x^{(x^2)}$ (c) $x^{(x^x)}$ (d) $x^{(x^{(x+1)})}$ (e) $x^{(x^{(x^x)})}$

10. A certain TV station has a logo which is a rotating cube in which one face has an A on it and the other five faces are blank. Originally the A -face is at the front of the cube as shown on the right. Then you perform the following sequence of three moves over and over: rotate the cube 90° around the vertical axis v , so that the front face moves to the left; then rotate the cube 90° around a horizontal axis h , so that the new front face moves down; then rotate the cube 90° around the vertical axis again, so that the new front face moves to the left. Suppose you perform this sequence of three moves a total of 1998 times. What will the front face look like when you have finished?



(a)



(b)



(c)



(d)

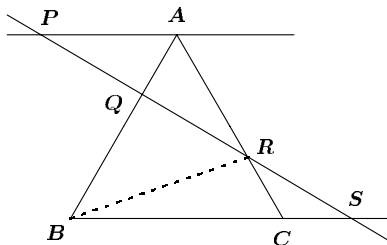


(e)

11. How many triples (x, y, z) of real numbers satisfy the simultaneous equations $x + y = 2$ and $xy - z^2 = 1$?

- (a) 1 (b) 2 (c) 3 (d) 4 (e) infinitely many

12. In the diagram, ABC is an equilateral triangle of side length 3 and PA is parallel to BS . If $PQ = QR = RS$, what is the length of BR ?



- (a) $\sqrt{6}$ (b) $\sqrt{6}$ (c) $\frac{3\sqrt{3}}{2}$ (d) $\sqrt{7}$ (e) none of these

13. Let a, b, c and d be the roots of $x^4 - 8x^3 - 21x^2 + 148x - 160 = 0$. What is the value of $\frac{1}{abc} + \frac{1}{abd} + \frac{1}{acd} + \frac{1}{bcd}$?

- (a) $-\frac{4}{37}$ (b) $-\frac{1}{20}$ (c) $\frac{1}{20}$ (d) $\frac{4}{37}$ (e) none of these

14. Wei writes down, in order of size, all positive integers b with the property that b and 2^b end in the same digit when they are written in base 10. What is the 1998th number in Wei's list?

- (a) 19974 (b) 19976 (c) 19994 (d) 19996 (e) none of these

15. Suppose that $x = 3^{1998}$. How many integers are there between $\sqrt{x^2 + 2x + 4}$ and $\sqrt{4x^2 + 2x + 1}$?

- (a) $3^{1998} - 2$ (b) $3^{1998} - 1$ (c) 3^{1998} (d) $3^{1998} + 1$ (e) $3^{1998} + 2$

16. The lengths of all three sides of a right triangle are positive integers. The area of the triangle is 120. What is the length of the hypotenuse?

- (a) 13 (b) 17 (c) 26 (d) 34 (e) none of these

Last number we gave the problems of the Mini demi-finale 1996 of the Olympiade Mathématique Belge. Somehow the last three problems were overlooked.

28. Lors d'un championnat de poursuite, deux cyclistes partent en même temps de deux points diamétralement opposés d'un vélodrome de 250 m de tours. Le vainqueur a rattrapé son rival après avoir parcouru 8 tours. Quel est le rapport de la vitesse moyenne du vainqueur à celle du perdant ?

- (a) $\frac{9}{8}$ (b) $\frac{8}{7}$ (c) $\frac{17}{16}$ (d) $\frac{16}{15}$ (e) $\frac{7}{8}$

29. Il était une fois ... deux fontaines et une citerne. La première fontaine mettait un jour entier à remplir la citerne, la seconde quatre. En combien de temps la citerne peut-elle être remplie par les deux fontaines coulant ensemble ?

- (a) 1 h 15 min (b) 8 h (c) 18 h (d) 19 h 12 min (e) 30 h

30. Laquelle des propositions suivantes est vraie dans tout carré ?

- (a) La longueur d'un côté est égale à la moitié de la longueur d'une diagonale.
 (b) La longueur d'un côté est égale à la racine carrée du périmètre.
 (c) La longueur d'un côté est supérieure aux deux tiers de la longueur d'une diagonale.
 (d) La longueur d'un côté est inférieure aux deux tiers de la longueur d'une diagonale.
 (e) La longueur d'un côté est égale à la longueur d'une diagonale multipliée par $\sqrt{2}$.

And now the answers.

1.	101	2.	e	3.	c	4.	e	5.	c
6.	c	7.	b	8.	64	9.	b	10.	e
11.	e	12.	b	13.	b	14.	c	15.	a
16.	c	17.	d	18.	d	19.	e	20.	315
21.	d	22.	c	23.	d	24.	7	25.	b
26.	e	27.	b	28.	d	29.	d	30.	c

That completes the *Skoliad Corner* for this issue. Send me your suitable contest materials, as well as suggestions for features for this *Corner*.