THE ACADEMY CORNER

No. 24

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Here is the second of four articles from the 1998 Canadian Undergraduate Mathematics Conference, held at the University of British Columbia in July 1998.

Abstracts • Résumés

Canadian Undergraduate Mathematics Conference 1998 — Part 2

Topological Censorship: A Theorem in Global General Relativity

Joel K. Erickson
University of British Columbia

Since the 1960's, modern differential geometry and algebraic topology have been used to study the global properties of space-times. In particular, these tools have been used to study the questions "what is the topology of our universe?" and "why don't we observe any non-trivial topology?" Neither the Einstein equations nor physical considerations provide grounds for restricting the range of possible spatial topologies; there is no reason to assume that spatial slices of our universe have one of the trivial topologies S^3 or $\mathbb{R}^3$. Friedman, Schleich, and Witt have proved that if a number of physically reasonable conditions are satisfied by space-time, then non-trivial topological structures cannot be probed; that is, a *topological censorship principle* holds. Following a brief introduction to causal structure, the theorem and its proof will be described. Current efforts to generalize the theorem to cosmological models will then be discussed.

Modern Answers to an Old Number Theory Question

Alexandru E. Ghîtză
McGill University

What integers can be expressed as a sum of two cubes of rational numbers? This simply stated problem, more than 350 years old, has been investigated by some of the

big names in mathematics: Fermat, Lagrange, Euler, Legendre, Dirichlet. Nonetheless, the question has only been solved in a few particular cases. We shall present some of these from two completely different points of view: that of classical number theory, illustrated by a 19th century theorem of Sylvester, and that of modern number theory, represented by the Heegner point construction which has been used recently to attack the problem.

Les partages d'entiers

Philippe Girard
Université du Québec à Montréal

Le problème général de la théorie additive des nombres consiste à déterminer le nombre de façons d'exprimer un entier naturel en une somme de ces derniers. Le partages d'entiers (en anglais "integer partitions") s'intéressent à ces décompositions dans la mesure où l'ordre des termes est sans importance. Le premier mathématicien à se pencher sur la question et qui apporta des résultats intéressant est Euler dans les années 1740. Dans cet exposé, une légère introduction à la théorie des partages d'entiers sera présentée à l'aide de quelques définitions, d'exemples, de théorèmes proposés par Euler et de démonstrations.

Variations sur quelques généralisations en analyse

Alexandre Girouard
Université de Montréal

L'analyse c'est vue, au cours du dernier siècle, completement transformée. Nous présentons ici quelques une des généralisations lui ayant donnée sa puissance actuelle ! Le matériel couvert ici est trés facilement disponible dans la littérature. Pour cette raison, ce document n'est presque rien de plus qu'une liste de référence.

Finite-Difference Approximations for Partial Differential Equations

Jacinthe Granger-Piché
Université de Montréal

Partial differential equations are used to represent a wide range of phenomena. In particular, they are well used to describe cloud formation. Often, analytical solutions of such equations cannot be found, so numerical techniques are needed. In this paper, we will present different finite-difference approximations applied to the advection and the heat equations, as well as their application to cloud formation.

Clifford Algebras and Spinors

Marco Gualtieri
McGill University

Clifford algebras are ideally suited for the study of rotations (for any signature metric). In fact, Clifford called his invention "Geometric Algebra."

I will describe the Real and Complex Clifford algebras, and the marvellous Bott 8-periodicity. Then I will give a general description of a *spinor*, and how it is used in physics to represent "spin - 1/2 particles" like the electron. I may finally discuss the mysterious coincidence called "Triality" which can be used to understand the *octonions*, the exceptional Lie groups, and other exotic animals.

For maximum enjoyment, you should know some linear algebra, (especially the Sylvester Inertia theorem for real symmetric bilinear forms).

Witten's Formulas for Symplectic Volumes of Moduli Spaces

Patrick Hayden
McGill University

This paper develops the geometric and algebraic tools necessary to understand Witten's calculation of the symplectic volumes of moduli spaces of flat connections over Riemann surfaces and then goes on to supply a detailed account of his argument. We summarize the basic constructions and theorems from differential geometry that are required for our investigation of moduli spaces before defining the Reidemeister torsion and proving some useful results for evaluating the torsion of complexes defined over surfaces. Next, we introduce the moduli space of flat connections on a Riemann surface, giving both the geometric construction and the equivalent representation-theoretic one. Our account ends with a proof of the equivalence of the symplectic volume and Reidemeister torsion and a calculation for the symplectic volume of the moduli space of flat connections over compact oriented surfaces of genus $g \geq 2$.

Reduced Decompositions of Permutations

Sylvie Hébert
Université du Québec à Montréal

After having recalled that the symmetric group is generated by adjacent transpositions, we will define reduced expressions (or decompositions) of permutations. The length of such a decomposition is the number of inversions of the permutation. We will show how to pass from a reduced decomposition to another, and how one can draw such a decomposition by a configuration of lines in the plane. Finally we will deduce the Coxeter presentation of the symmetric group.

A New Characterization of Topology?

Patrick Ingram
Simon Fraser University

In most classical texts on topology we are given several equivalent methods of representing topological spaces : by open sets, by closed sets, by an interior function, and by a closure function. We are also given methods of representing a topological space in terms of another topological space and a function between them that we wish to be continuous. This author, however, was not able to find a characterization from sequence convergence and/or clustering, and as such this paper explores these possibilities. This paper contains elementary definitions to make it possible for a student with a minimal pure math background to read it.