BOOK REVIEWS

Edited by ANDY LIU

Mathematical Challenge, by **Tony Gardiner**,
published by Cambridge University Press, 1996,
ISBN# 0-521-55875-1, softcover, 138+ pages.

More Mathematical Challenges, by **Tony Gardiner**,
published by Cambridge University Press, 1997,
ISBN# 0-521-58568-6, softcover, 140+ pages.

*Reviewed by **Ted Lewis**, University of Alberta.*

These two books are companion volumes which cover the recent phenomenal development in mathematics competitions in the United Kingdom. Although the country has a long and distinguished history in this endeavour, featuring the famed Cambridge Tripos, it was not until the late 1980's when a popularization movement, under the leadership of the author Tony Gardiner, made it a truly national event. This grassroots approach has nurtured the young talents into a force to be reckoned with consistently in the International Mathematical Olympiad.

The first book contains the problems for the UK Schools Mathematical Challenge Papers from 1988 to 1993. The target audience are children aged 12 to 14. Each paper consists of 25 multiple-choice questions, of which the first 15 are relatively straightforward. The paper is to be attempted in one hour, and students are not expected to finish it. Random guessing is discouraged, and calculators are forbidden. Answers, statistics and brief comments are given, but the reader has to work out the solutions.

The second book contains the problems for the UK Junior Mathematical Olympiad from 1989 to 1995. The target audience is children aged 11 to 15. Except in 1989, each paper consists of 10 problems in Section A and 6 problems in Section B. The 1989 paper consists of 13 problems, and may be regarded as a long Section B. The following paragraph is quoted from page 2 of the book.

> *Section A problems are direct, "closed" problems, each requiring a specific calculation and having a single numerical answer. Section B problems are longer and more "open". Thus, while the final mathematical solution is often quite short, and should involve a clear claim, followed by a direct deductive calculation or proof, there will generally be a preliminary phase of exploration and conjecture, in which one tries to sort out how to tackle the problem.*

There is a section titled "Comments and hints" and another titled "Outline solutions". Thus a student who is frustrated by a particular problem has several recourses for assistance. The outline solutions are precisely that, with gaps for the reader to fill in, but revealing enough details to guide a diligent student to the full solution.

We now present some sample problems.

1988/14.
Weighing the baby at the clinic was a problem. The baby would not keep still and

caused the scales to wobble. So I held the baby and stood on the scales while the nurse read off 79 kg. Then the nurse held the baby while I read off 69 kg. Finally I held the nurse while the baby read off 137 kg. What is the combined weight of all three (in kg)?

(a) 142 (b) 147 (c) 206 (d) 215 (e) 284

1993/18.
Sam the super snail is climbing a vertical gravestone 1 metre high. She climbed at a steady speed of 30 cm per hour, but each time the church clock strikes, the shock causes her to slip down 1 cm. The clock only strikes the hours, so at 1 o'clock she would slip back 1 cm, at 2 o'clock she would slip back 2 cm and so on. If she starts to climb just after the clock strikes 3 pm, when will she reach the top?

(a) 3:50 pm (b) 6:20 pm (c) 6:50 pm (d) 7:04 pm (e) 7:20 pm

1994/A6.
A normal duck has two legs. A lame duck has one leg. A sitting duck has no legs. Ninety nine ducks have a total of 100 legs. Given that there are half as many sitting ducks as normal ducks and lame ducks put together, find the number of lame ducks.

1995/B6.
I write out the numbers 1, 2, 3, 4 in a circle. Starting at 1, I cross out every second integer till just one number remains: 2 goes first, then 4, leaving 1 and 3; 3 goes next, leaving 1 — so "1" is the last number left. Suppose I start with 1, 2, 3, ..., n in a circle. For which values of n will the number "1" be the last number left?

Comments and hints for 1995/B6.
The question "For which values of $n \ldots$?" is more complicated than it looks. It is not enough to answer "$n = 4$" (even though $n = 4$ works), since we clearly want to know *all* possible values of n that work. Hence a solution must not only show that certain values of n work, but must also somehow prove that *no other values of n could possibly work*.

Outline solution of 1995/B6.
The easy step is to realize that, if n is odd (and ≥ 3) then 1 will be crossed out at the start of the second circuit. Hence, for 1 to survive, either $n = 1$ or n must be e⋆e⋆ (say $n = 2m$). Suppose $n = 2m$. Then at the start of the second circuit, there remain exactly m numbers 1, 3, 5, ..., $2m - 1$. For 1 to survive next time it is essential that either $m = 1$ (so $n = 2$) or m must be e⋆e⋆ (say $m = 2p$). Continuing in this way we see that 1 will be the last uncrossed number if and only if n is a ⋆o⋆e⋆ of ⋆⋆o. [Ed. here ⋆ indicates some number.]

As a bonus, the first book provides 420 additional questions, and the second 40 Section A problems and 20 Section B problems. For one reason or another, these did not make it into the competition papers, but are nevertheless excellent questions.

A lot of thought has gone into the planning of these two books. They are to be *done actively* rather than read passively. The problems are the important features, not the solutions. Particularly valuable are the author's comments on how things are done and *why*. This dynamic package is a must for anyone interested in mathematics competitions for youngsters.