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### SYNOPSIS

#### 129 The Academy Corner: No. 18    *Bruce Shawyer*

Featuring two solutions to the 1997 Memorial University Undergraduate Mathematics Competition.

#### 131 The Olympiad Corner: No. 189    *R.E. Woodrow*

Featuring Peru's Selection Test for the XII Iberoamerican Olympiad; the third and fourth grade problems of the 38<sup>th</sup> Mathematics Competition for Secondary School Students of the Republic of Slovenia; the VIII Nordic Mathematical Contest; comments and solutions related to the February 1997 number of the corner; two alternate solutions to problems of the Sixth Irish Mathematical Olympiad; and solutions to problems of the Latvian 44 Mathematical Olympiad.

#### 143 Book Review    *Andy Liu*

*Vita Mathematica*, edited by Ronald Calinger

Reviewed by *Maria de Losada*

#### 145 Sum of powers of a finite sequence: a geometric approach

*William O.J. Moser*

In this note we give a geometric-combinatoric derivation of a formula for the sum of  $r^{\text{th}}$  powers of a finite positive integer sequence:

$$a_1^r + a_2^r + \cdots + a_n^r = \sum_{\ell=1}^r \sum_{i=1}^n \binom{a_i}{\ell} \mu(r, \ell), \quad (1)$$

where

$$\binom{n}{k} = \begin{cases} \frac{n!}{k!(n-k)!}, & \text{if } 0 \leq k \leq n, \\ 0, & \text{if } k > n \geq 1, \end{cases}$$

and the numbers  $\mu(r, \ell)$  are determined by the recurrence

$$\begin{aligned} \mu(r, 1) &= 1, \quad r = 1, 2, 3, \dots; & \mu(1, \ell) &= 0, \quad \ell = 2, 3, \dots; \\ \mu(r, \ell) &= \ell(\mu(r-1, \ell-1) + \mu(r-1, \ell)), & r &\geq 2, \ell \geq 2. \end{aligned} \quad (2)$$

The derivation parallels, simplifies and generalizes the proof given in [1], and seems to be more elementary than proofs given in [2] and [3].

1 Moser, W., Sums of  $d^{\text{th}}$  powers. *Math. Gazette* 75 (1991) 332.

2 Paul, J.L., On the sum of the  $k^{\text{th}}$  powers of the first  $n$  integers.  
*Amer. Math. Monthly* **78** (1971) 271–273. MR 43 #4092.

3 Wagner, C., Combinatorial proofs of formulas for power sums.  
*Arch. Math. (Basel)* **68** (1997), no. 6, 464–467.

148 The Skoliad Corner: No. 29 *R. E. Woodrow*

Featuring the 1996 Alberta High School Mathematics Competition, Part I; and the solutions to the British Columbia Colleges Junior High School Mathematics Contest Preliminary Round (1997).

158 Mathematical Mayhem

158 Mayhem Problems

159 High School Problems

159 Advanced Problems

160 Challenge Board Problems

161 Tips on Inequalities

*Naoki Sato*

168 Riveting Properties of Pascal's Triangle

*Richard Hoshino*

173 Swedish Mathematics Olympiad, 1985 Qualifying Round.

175 Problems: 2306, 2326–2337

This month's "free sample" is:

2337. Proposed by Iliya Bluskov, Simon Fraser University, Burnaby, BC.

Let  $F(1) = \left\lceil \frac{n^2 + 2n + 2}{n^2 + n + 1} \right\rceil$ , and, for each  $i > 1$ , let

$$F(i) = \left\lceil \frac{n^2 + 2n + i + 1}{n^2 + n + i} F(i - 1) \right\rceil.$$

Find  $F(n)$ .

178 Solutions: 2219–2222, 2224–2226, 2229–2230