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## SYNOPSIS



In this note we give a geometric-combinatoric derivation of a formula for the sum of $r^{\text{th}}$ powers of a finite positive integer sequence:

$$a_1^r + a_2^r + \cdots + a_n^r = \sum_{\ell=1}^{r}\sum_{i=1}^{n}\binom{a_i}{\ell}\mu(r,\ell)\,, \qquad (1)$$

where

$$\binom{n}{k} = \begin{cases} \frac{n!}{k!(n-k)!}, & \text{if } 0 \le k \le n\,, \\ 0, & \text{if } k > n \ge 1\,, \end{cases}$$

and the numbers $\mu(r,\ell)$ are determined by the recurrence

$$\begin{aligned} \mu(r,1) &= 1,\ \ r = 1,2,3,\cdots;\quad \mu(1,\ell) = 0,\ \ \ell = 2,3,\cdots; \\ \mu(r,\ell) &= \ell(\mu(r-1,\ell-1) + \mu(r-1,\ell)),\ r \ge 2,\ \ell \ge 2\,. \end{aligned} \qquad (2)$$

The derivation parallels, simplifies and generalizes the proof given in [1], and seems to be more elementary than proofs given in [2] and [3].

1 Moser, W., Sums of $d^{\text{th}}$ powers. *Math. Gazette* **75** (1991) 332.