

THE ACADEMY CORNER

No. 18

Bruce Shawyer

All communications about this column should be sent to Bruce Shawyer, Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. A1C 5S7

Memorial University Undergraduate Mathematics Competition

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We present in this issue, two solutions from Bob Prielipp, University of Wisconsin–Oshkosh, Wisconsin, USA, to the above competition, which was printed in Academy Corner No. 15 [1997: 449].

2. The surface area of a closed cylinder is twice the volume. Determine the radius and height of the cylinder given that the radius and height are both integers.

Solution.

Let r be the radius and h be the height of the closed cylinder, where r and h are both positive integers.

If the surface area of the closed cylinder is twice the volume, then

$$2\pi rh + 2\pi r^2 = 2\pi r^2 h.$$

It follows that $h + r = rh$. Thus

$$(r - 1)(h - 1) = rh - r - h + 1 = 0 + 1 = 1.$$

Hence $r - 1 = 1$ and $h - 1 = 1$ (since r and h are both positive integers), so that $r = 2$ and $h = 2$.

It is easily checked that if $r = 2$ and $h = 2$, then the surface area of the closed cylinder is twice the volume.

3. Prove that

$$1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{n^2} < 2.$$

Solution. [Slightly shortened by the editor.]

We use mathematical induction to prove, for each positive integer n ,

$$1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{n^2} < 2 - \frac{1}{n}.$$

The result clearly holds for $n = 1$.

Assume that the result holds for $n = k$. Then

$$\begin{aligned} 1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{k^2} + \frac{1}{(k+1)^2} &< \left(2 - \frac{1}{k}\right) + \frac{1}{(k+1)^2} \\ &= 2 - \frac{(k+1)^2 - k}{k(k+1)^2} = 2 - \frac{k^2 + k + 1}{k(k+1)^2} \\ &< 2 - \frac{k(k+1)}{k(k+1)^2} = 2 - \frac{1}{k+1}. \end{aligned}$$

Hence, by induction, the result holds.

Equality holds if and only if $n = 1$.

Advance Notice

At the summer 1999 meeting of the Canadian Mathematical Society, to be held in St. John's, Newfoundland, there will be a Mathematics Education Session on the topic "What Mathematics Competitions do for Mathematics".

Invited speakers include Edward Barbeau, Toronto; Tony Gardner, Birmingham, England; Ron Dunkley, Waterloo; and Rita Janes, St. John's. Anyone interested in giving a paper at this session should contact one of the organizers, Bruce Shawyer or Ed Williams, at the Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, Newfoundland, Canada.

email addresses:

bshawyer@math.mun.ca
ewilliam@math.mun.ca