THE SKOLIAD CORNER

No. 21

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This number we give the problems of the Manitoba Mathematical Contest for 1995. This is a two hour contest aimed primarily at grade 12 students, and sponsored by the Actuaries Club of Winnipeg, The Manitoba Association of Mathematics Teachers, The Canadian Mathematical Society and The University of Manitoba. My thanks go to Diane and Roy Dowling, organizers of the contest for supplying us with it.

THE MANITOBA MATHEMATICAL CONTEST 1995

For Students in Grade 12

Wednesday, February 22, 1995 — Time: 2 hours

1. (a) If a and b are real numbers such that $a+b=3$ and $a^2+ab=7$ find the value of a.

(b) Noriko's average score on three tests was 84. Her score on the first test was 90. Her score on the third test was 4 marks higher than her score on the second test. What was her score on the second test?

2. (a) Find two numbers which differ by 3 and whose squares differ by 63.

(b) Find the real number which is a root of the equation

$$27(x-1)^3+8=0.$$

3. (a) Two circles lying in the same plane have the same centre. The radius of the larger circle is twice the radius of the smaller circle. The area of the region between the two circles is 7. What is the area of the smaller circle?

(b) The area of a right triangle is 5. Also, the length of the hypotenuse of this triangle is 5. What are the lengths of the other two sides?

4. (a) The parabola whose equation is $8y=x^2$ meets the parabola whose equation is $x=y^2$ at two points. What is the distance between these two points?

(b) Solve the equation $3x^3+x^2-12x-4=0$.

5. (a) Find the real number a such that $a^4-15a^2-16=0$ and $a^3+4a^2-25a-100=0$.

(b) Find all positive numbers x such that $x^{x\sqrt{x}}=(x\sqrt{x})^x$.

6. If x, y and z are real numbers prove that

$$(x|y| - y|x|)\,(y|z| - z|y|)\,(x|z| - z|x|) = 0.$$

7. x and y are integers between 10 and 100. y is the number obtained by reversing the digits of x. If $x^2 - y^2 = 495$ find x and y.

8. Three points P, Q and R lie on a circle. If $PQ = 4$ and $\angle PRQ = 60°$ what is the radius of the circle?

9. Three points are located in the finite region between the x-axis and the graph of the equation $2x^2 + 5y = 10$. Prove that at least two of these points are within a distance 3 of each other.

10. Three circles pass through the origin. The centre of the first circle lies in the first quadrant, the centre of the second circle lies in the second quadrant, and the centre of the third circle lies in the third quadrant. If P is any point that is inside all three circles, show that P lies in the second quadrant.

Last number we gave the problems of the Mathematical Association National Mathematics Contest 1994 from the United Kingdom. Here are the answers.

1.	C	**2.**	E	**3.**	C	**4.**	D	**5.**	E
6.	B	**7.**	A	**8.**	D	**9.**	B	**10.**	B
11.	B	**12.**	E	**13.**	E	**14.**	A	**15.**	A
16.	C	**17.**	D	**18.**	C	**19.**	B	**20.**	B
21.	C	**22.**	B	**23.**	E	**24.**	B	**25.**	E

That completes the Skoliad Corner for this issue. I need suitable contest materials and welcome your suggestions for the evolution of this feature.