

THE SKOLIAD CORNER

No. 20

R.E. Woodrow

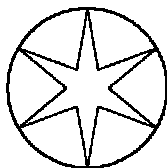
This issue we give the problems of the Mathematical Association National Mathematics contest, written November 18, 1994. The contest was written by about 30,000 students in the United Kingdom. My thanks go to Tony Gardiner, School of Mathematics, University of Birmingham for sending me the contest.

THE MATHEMATICAL ASSOCIATION NATIONAL MATHEMATICS CONTEST 1994

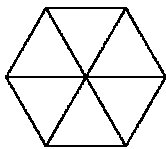
Friday, November 18, 1994 — Time: 90 minutes

1. The average of x and $8x$ is 18. What is the value of x ?
- A. $1\frac{1}{2}$ B. 2 C. 4 D. $4\frac{1}{2}$ E. 9.

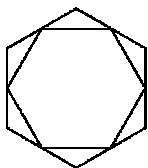
2. Which of the following does not have six lines of symmetry?



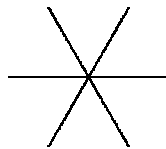
A. fig 2a



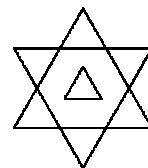
B. fig 2b



C. fig 2c



D. fig 2d



E. fig 2e.

3. I write out the numbers from 1 up to 30 *in words*. If N denotes the number of times I write the letter “n”, M denotes the number of times I write the letter “m”, and C denotes the number of times I write the letter “c”, then $N + M + C$ equals

- A. 27 B. 28 C. 29 D. 30 E. 31.

4. Which of the following five numbers has a prime factor in common with exactly one of the other four numbers?

- A. 91 B. 52 C. 39 D. 35 E. 24.

5. $ABCD$ is a quadrilateral with $AB = AD = 25$ cm, $CB = CD = 52$ cm and $DB = 40$ cm. How long is AC in cm?

- A. 32.5 B. 48 C. 52 D. 60 E. 63.

6. The number of pounds of pickled peppers that Peter Piper purchased for £59 is equal to the number of pounds Peter would pay for two hundred and thirty six pounds of peppers. How much would he pay for twenty pounds of pickled peppers?

- A. £5 B. £10 C. £20 D. £40 E. £80.

7. Which expression has the smallest value when $x = -0.5$?

- A. $2^{1/x}$ B. $\frac{-1}{x}$ C. $\frac{1}{x^2}$ D. 2^x E. $\frac{1}{\sqrt{-x}}$.

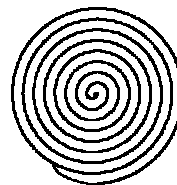
8. Over an average lifetime in the UK, roughly how many times does a person's heart beat?

- A. 4×10^7 B. 5×10^7 C. 2×10^8 D. 3×10^9 E. 2×10^{10} .

9. What is the sum of the reciprocals of the first six triangular numbers 1, 3, 6, 10, etc.?

- A. 10 B. $\frac{12}{7}$ C. 56 D. $\frac{3}{2}$ E. $\frac{49}{21}$.

10. A rope 15 m long and 5 cm in diameter is coiled in a flat spiral as shown. What is the best estimate for the diameter of the "circle" (in cm)?

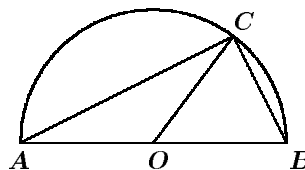


- A. 10 B. 100 C. 150 D. 200 E. 300.

11. If $a \otimes b = \frac{(ab+a+b+1)}{a}$, then $19 \otimes 94$ equals

- A. 95 B. 100 C. 208 D. 1882 E. 1994.

12. The diagram shows a semicircle with radius 1 cm and with centre O . If C is an arbitrary point on the semicircle, which of the following statements may be false?



- A. $\angle ACB$ is a right angle. B. $\triangle OAC$ is isosceles.
 C. the area of $\triangle ABC$ is $\leq 1 \text{ cm}^2$ D. $\triangle AOC$ is equal in area to $\triangle OBC$
 E. $AO^2 + OB^2 = AC^2 + BC^2$.

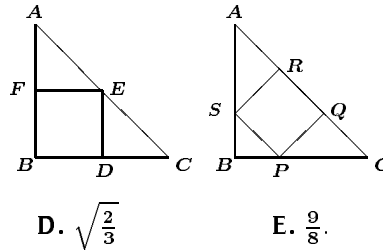
13. A giant marrow in my garden weighed 50 pounds and was 98% water. Suppose that during a rainy day it absorbed water so that it became 99% water. What would its new weight be (in pounds)?

- A. 50.01 B. 50.5 C. 98 D. 99 E. 100.

14. A solid cuboid has edges of lengths a , b , c . What is its surface area?

- A. $(a + b + c)^2 - (a^2 + b^2 + c^2)$ B. abc C. $2(a^2 + b^2 + c^2)$
D. $(a + b + c)^2$ E. $ab + bc + ca$.

15. Given two copies of an isosceles right-angled triangle ABC , squares $BDEF$ and $PQRS$ are inscribed in different ways as shown. What is the ratio $\frac{\text{area } PQRS}{\text{area } BDEF}$?



- A. $\frac{8}{9}$ B. $\frac{2}{3}$ C. 1 D. $\sqrt{\frac{2}{3}}$ E. $\frac{9}{8}$.

16. What is the last digit of $1994^{(1995+1996+1997+1998+1999+2000)}$?

- A. 0 B. 2 C. 4 D. 6 E. 8.

17. When two dice are thrown the probability that the total score is a multiple of 2 is $\frac{1}{2}$. For how many other values of n is it true that, when two dice are thrown, the probability that the total score is a multiple of n is equal to $\frac{1}{n}$?

- A. 1 B. 2 C. 3 D. 4 E. 5.

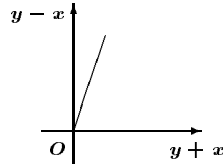
18. How many digits are there in the smallest number which is composed entirely of fives (eg. 5555) and which is divisibly by 99?

- A. 9 B. 10 C. 18 D. 36 E. 45.

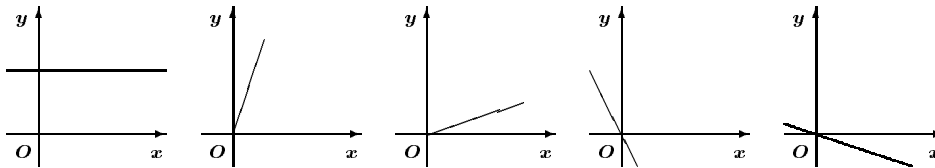
19. The price of a secondhand car is displayed (in pounds) on four cards on the windscreen. Each card shows one digit. If the card with the thousands digit blew off in the wind, the apparent price of the car would drop to one forty-ninth of the intended value. What number is on that card?

- A. 5 B. 6 C. 7 D. 8 E. 9.

20. The graph of $y - x$ against $y + x$ is as shown.



The same scale has been used on each axis. Which of the following shows the graph of y against x ?



- A. fig 20a B. fig 20b C. fig 20c D. fig 20d E. fig 20e.

21. Which is smallest?

- A. $5 + 6\sqrt{7}$ B. $7 + 6\sqrt{5}$ C. $6 + 5\sqrt{7}$ D. $7 + 5\sqrt{6}$ E. $6 + 7\sqrt{5}$.

22. A train leaves London at 0600 and arrives in Newcastle at 0930. Another train leaves Newcastle at 0700 and arrives in London at 0930. If both used the same route and each travelled at a constant speed, at what time would they meet?

- A. $0757\frac{1}{2}$ B. $0802\frac{1}{2}$ C. $0807\frac{1}{2}$ D. $0827\frac{1}{2}$ E. more information required.

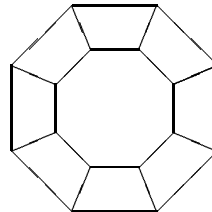
23. The triangle ABC has a right-angle at A . The hypotenuse BC is trisected at M and at N so that $BM = MN = NC$. If $AM = x$ and $AN = y$, then MN is equal to

- A. $\frac{x+y}{2}$ B. $\frac{\sqrt{(y^2-x^2)}}{2}$ C. $\sqrt{(y^2-x^2)}$ D. $\frac{\sqrt{(x^2+y^2)}}{3}$ E. $\sqrt{\frac{x^2+y^2}{5}}$.

24. If Susan drives to work at x mph she will be one minute late; if she drives at y mph she will be one minute early. How far does she drive to work (in miles)?

- A. $\frac{yx}{30(y-x)}$ B. $\frac{2yx}{y-x}$ C. $\frac{x+y}{y-x}$ D. $\frac{x+y}{2}$ E. $\frac{x+y}{60(y-x)}$.

25. The octagonal figure is obtained by fitting eight congruent isosceles trapezia together. If the three shorter sides of each trapezium have length 1, how long is each outer edge?



A. $1 + \sqrt{2}$

B. $\frac{1+\sqrt{2}}{2}$

C. $\sqrt{2}$

D. 2

E. $1 + \sqrt{2 - \sqrt{2}}$

Last number we gave the 1994 Nat West U.K. Junior Mathematical Challenge. It was written Tuesday, April 26, 1994. Here are the answers.

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|--------|-------|-------|-------|-------|
| 1. 455 | 2. B | 3. E | 4. A | 5. A |
| 6. B | 7. B | 8. D | 9. D | 10. A |
| 11. B | 12. E | 13. A | 14. B | 15. E |
| 16. E | 17. C | 18. C | 19. D | 20. E |
| 21. A | 22. D | 23. D | 24. C | 25. C |

That completes the Skoliad Corner for this issue. Send me your suitable contests and solutions. Also send me any suggestions for improvement of this feature.

