BOOK REVIEWS

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Leningrad Mathematical Olympiads 1987–1991, by Dmitry Fomin and Alexey Kirichenko,
published by MathPro Press, Westford, MA, USA, 1994.
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The regulations of the International Mathematical Olympiad (IMO) stipulate that a country can send a team to the IMO consisting of at most six students. Still, there was a *city* in the 1965 IMO which participated with *nine* students: five in the Russian team, two in the Israeli team, and one in each of the USA and German teams. This city was Leningrad, now known by its old name St. Petersburg again.

If a city has such an output of talented young mathematicians, you can imagine that the city olympiad organized there should be of an exceptionally high quality. And so it is. Let me briefly describe the system of the Leningrad Olympiads. Until 1989 Soviet schools comprised grades 1–10, and the competition was held for 5–10 graders. Since that time the Russian schools comprise grades 1–11, and the competition is held for 6–11 graders. The competition is organized in four rounds. The first round is at the school level in December and January and the second round held at the regional level (that is, in the 22 regions of Leningrad) in February, some 10,000 to 12,000 students taking part in the second round. Both of these rounds are written examinations. It is important to emphasize this, because the last two rounds — quite unusually for a mathematics competition — are oral examinations. This requires of course a huge number of able and devoted jurors, and few countries — let alone cities — can produce the number of mathematicians necessary for this task. In the third round (called "main round") about 90–130 students participate in each grade. In the final round (called somewhat misleadingly "elimination round" — this term normally means the initial phase of a competition, not the final one) which is held only for 9–11 (earlier 8–10) graders some 100 students take part altogether, although in 1991 there were only 34. The main round is held in February or March, and the elimination round in March. In the period 1962–1983 and also in 1991 the elimination round was used only to pick the city team at the All-Union Olympiad (this might explain the strange name), while in the period 1984–1990 the result of the competition was decided by the elimination round (not by the main round).

A few words about the earlier history of the Leningrad (and other) Olympiads in the Soviet Union. The Leningrad Olympiad started in 1934 (the Moscow Olympiad one year later). The All-Russia Olympiad started in 1961 and the All-Union (i.e. Soviet Union) Olympiad in 1967. These are respectable ages, although there are a few national mathematics contests which are much older (notably the Hungarian Eötvös — later Kürschák — competitions which started in 1894). But I fully agree with the authors that the Leningrad competition is quite unique in being an oral one. On the other hand I must contest another statement of the authors, namely that the Leningrad competition would be quite unique in having only new and original problems. In fact the majority of the competitions I know of have new and original problems.

Besides the main part containing the problems and solutions the book contains useful appendices: statistical data about the number of participants, the names of each year's winners, a glossary and the names of the authors of each problem. Also, there are valuable comments on the book by the publisher, Stanley Rabinowitz, and a very good evaluation of the reasons of the excellence of Soviet mathematics by Mark Saul.

To give a sample of the problems (and solutions) I give you two examples from the book. Both come from the 1991 Olympiad and were given to grade 11 students.

Problem A black pawn is placed in the top right square of an 8×8 chess-board. One may place a white pawn on any empty square of the board, having repainted all pawns in adjacent squares so that black pawns become white, and vice versa. (Two squares are called adjacent if they have a common vertex.) Can one place pawns in this manner so that all 64 squares of the board would be filled with white pawns?

Solution Connect the centres of each pair of adjacent free squares by a segment. Then, when we place the next white pawn on some square of the board, we erase all segments from the centre of this square. Thus, the number of erased segments coincides each time with the number of free adjacent squares for the square on which each white pawn is placed. To make a certain pawn be white at the end, it must be placed on a square having an even number of free adjacent squares because the number of repaintings for each pawn is equal to the number of pawns that will be placed on adjacent squares after this one. So, we conclude that each time we have to erase an even number of segments. But this is impossible because the initial number of segments is odd (we have lost at the beginning three segments caused by the black pawn in the corner) and the final number is even — it is equal to zero. Thus, we see that in the final position there is at least one black pawn.

The next problem has a rather unusual solution.

Problem One may perform the following two operations on a natural number:

(a) multiply it by any natural number;

(b) delete zeros in its decimal representation.

Prove that for any natural number n, one can perform a sequence of these operations that will transform n to a one-digit number.

Solution We use the following fact:

Lemma For any integer n that is not divisible by 2 or 5 one can find a number consisting of digits 1 only that is a multiple of n.

Proof Consider n numbers $1, 11, \dots, 11\dots1$. If one of these numbers is a multiple of n, we are done. Otherwise, their remainders modulo n can have $n-1$ possible values $1, \dots, n-1$, and therefore (by the pigeonhole principle) at least two of those numbers have the same remainders modulo n. Then their difference is divisible by n and looks like $11\dots1 \cdot 10^k$. But n is coprime with 10, and we conclude that the first factor is divisible by n.

Now let n be an arbitrary natural number. Multiplying n by 2 and 5 and deleting zeros, we can transform n to a number that is coprime with 10. Then multiplying the result by the appropriate number, we can obtain a number containing only digits 1 in its decimal representation (we use the assertion of the lemma). Now the chain of the following operations leads to the desired result:

(a) Multiplying by 82, we obtain the number $911\dots102$.

(b) Delete zero in the last obtained number and multiply it by 9. This gives the number $8200\dots08$, which is transformed into 828.

(c) $828 \cdot 25 = 20700$.

(d) $27 \cdot 4 = 108$.

(e) $18 \cdot 5 = 90$, and we can obtain the single-digit number 9.

All in all this book is very well written, full of interesting problems and I warmly recommend it to anyone interested in mathematical competitions, or just in nice problems.