

THE ACADEMY CORNER

No. 9

Bruce Shawyer

All communications about this column should be sent to Bruce Shawyer, Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. A1C 5S7

In this issue, we feature a university undergraduate mathematics competition. We invite your solutions, especially from university students. Please send me your nice solutions.

Undergraduate Mathematics Competition

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Answer as many questions as you can. Complete solutions carry more credit than scattered comments about many problems.

1. If n is any integer, show that $n^5 - n$ is divisible by 5.
2. A line l with slope $m = 2$ cuts the parabola $y^2 = 8x$ to form a chord. Find the equation of l if the midpoint of the chord lies on $x = 4$.
3. Show that three tangents can be drawn from the origin to the curve given by

$$y = x^3 - 13x^2 + 10x - 36.$$

4. Prove that $\binom{n}{0}^2 + \binom{n}{1}^2 + \dots + \binom{n}{n}^2 = \binom{2n}{n}$, for all positive integers n .
5. Show that

$$\int_0^{\pi/2} \frac{\sin^{13} x}{\sin^{13} x + \cos^{13} x} dx = \frac{\pi}{4}.$$

6. In triangle ABC , $\angle B = 3\angle A$.
If the sides opposite to the angles A, B, C have lengths a, b, c , respectively, prove that

$$ac^2 = (b - a)^2 (b + a).$$

