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SYNOPSIS

241 Dissecting Squares into Similar Rectangles

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Karl Scherer and Martin Gardner have proposed the following three-part problem. Cut a square into three similar pieces, where

(1) all three are congruent;

(2) exactly two are congruent;

(3) no two are congruent.

Note that (2) really consists of two sub-problems, where the congruent pieces are

(2a) smaller than the third;

(2b) larger than the third.

All their solutions to (2) are to (2a). It appears that, alas, (2b) is not to be. Also considered in Scherer and Gardner's note are related dissections of an equilateral triangle, but they do not concern us here.

We generalize the problem of Scherer and Gardner for the square as follows. Given any integer $m > 1$ and any of its 2^{m-1} compositions, or ordered partitions, $m = a_1 + a_2 + \cdots + a_n$, dissect a square into m similar pieces so that there are a_1 congruent pieces of the largest size, a_2 congruent pieces of the next largest size, and so on. In the original problem, $m = 3$ and the compositions are: (1) 3; (2a) $1 + 2$; (2b) $2 + 1$; (3) $1 + 1 + 1$.

Our main result is that the dissection problem always admits a solution using rectangular pieces if and only if the composition is not of the form $k + 1$, where k is any positive integer. These solvable cases are covered by two constructions which are only slightly different.

249 The Skoliad Corner: No. 16 *R.E. Woodrow*

Featuring the 1995 Saskatchewan Senior Mathematics Contest and the solutions to the 1996 American Invitational Mathematics Examination.

This month's "free sample" is:

2175. *Proposed by Christopher J. Bradley, Clifton College, Bristol, UK.*

The fraction $\frac{1}{6}$ can be represented as a difference in the following ways:

$$\frac{1}{6}=\frac{1}{2}-\frac{1}{3};\quad \frac{1}{6}=\frac{1}{3}-\frac{1}{6};\quad \frac{1}{6}=\frac{1}{4}-\frac{1}{12};\quad \frac{1}{6}=\frac{1}{5}-\frac{1}{30}.$$

In how many ways can the fraction $\frac{1}{2175}$ be expressed in the form

$$\frac{1}{2175}=\frac{1}{x}-\frac{1}{y},$$

where x and y are positive integers?