# THE SKOLIAD CORNER

## No. 12

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This issue we feature Part I of the 1995–96 Alberta High School Mathematics Competition, written November 21, 1995. My thanks go to Professor T. Lewis, the University of Alberta for forwarding me a copy. Students have 90 minutes to complete their contest. It is mostly written by students in Grade XII, but has often been won by students in earlier grades.

## ALBERTA HIGH SCHOOL MATHEMATICS COMPETITION

### Part I

**November 21, 1995 (Time: 90 minutes)**

**1.** A circle and a parabola are drawn on a piece of paper. The number of regions they divide the paper into is at most

**A.** 3 **B.** 4 **C.** 5 **D.** 6 **E.** 7.

**2.** The number of different primes $p > 2$ such that $p$ divides $71^2 - 37^2 - 51$ is

**A.** 0 **B.** 1 **C.** 2 **D.** 3 **E.** 4.

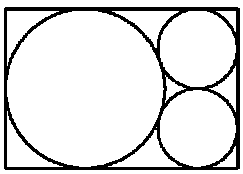
**3.** Suppose that your height this year is 10% more than it was last year, and last year your height was 20% more than it was the year before. By what percentage has your height increased during the last two years?

**A.** 30 **B.** 31 **C.** 32 **D.** 33 **E.** none of these.

**4.** Multiply the consecutive even positive integers together until the product $2 \cdot 4 \cdot 6 \cdot 8 \cdots$ becomes divisible by 1995. The largest even integer you use is

**A.** between 1 and 21 **B.** between 21 and 31
**C.** between 31 and 41 **D.** bigger than 41
**E.** non-existent, since the product never becomes divisible by 1995.

**5.** A rectangle contains three circles as in the diagram, all tangent to the rectangle and to each other. If the height of the rectangle is 4, then the width of the rectangle is




**A.** $3 + 2\sqrt{2}$ **B.** $4 + \frac{4\sqrt{2}}{3}$ **C.** $5 + \frac{2\sqrt{2}}{3}$
**D.** 6 **E.** $5 + \sqrt{10}$.

**6.** Mary Lou works a full day and gets her usual pay. Then she works some overtime hours, each at $150\%$ of her usual hourly salary. Her total pay that day is equivalent to $12$ hours at her usual hourly salary. The number of hours that she usually works each day is

**A.** $6$ **B.** $7.5$ **C.** $8$ **D.** $9$ **E.** not uniquely determined by the given information.

**7.** A fair coin is tossed $10,000$ times. The probability $p$ of obtaining at least three heads in a row satisfies

**A.** $0 \leq p < \frac{1}{4}$ **B.** $\frac{1}{4} \leq p < \frac{1}{2}$ **C.** $\frac{1}{2} \leq p < \frac{3}{4}$ **D.** $\frac{3}{4} \leq p < 1$ **E.** $p = 1$.

**8.** In the plane, the angles of a regular polygon with $n$ sides add up to less than $n^2$ degrees. The smallest possible value of $n$ satisfies:

**A.** $n < 40$ **B.** $40 \leq n < 80$ **C.** $80 \leq n < 120$ **D.** $120 \leq n < 160$ **E.** $n \geq 160$.

**9.** A cubic polynomial $P$ is such that $P(1) = 1$, $P(2) = 2$, $P(3) = 3$ and $P(4) = 5$. The value of $P(6)$ is

**A.** $7$ **B.** $10$ **C.** $13$ **D.** $16$ **E.** $19$.

**10.** The positive numbers $x$ and $y$ satisfy $xy = 1$. The minimum value of $\frac{1}{x^4} + \frac{1}{4y^4}$ is

**A.** $\frac{1}{2}$ **B.** $\frac{5}{8}$ **C.** $1$ **D.** $\frac{5}{4}$ **E.** no minimum.

**11.** Of the points $(0,0)$, $(2,0)$, $(3,1)$, $(1,2)$, $(3,3)$, $(4,3)$ and $(2,4)$, at most how many can lie on a circle?

**A.** $3$ **B.** $4$ **C.** $5$ **D.** $6$ **E.** $7$.

**12.** The number of different positive integer triples $(x, y, z)$ satisfying the equations

$$x^2 + y - z = 100 \quad \text{and} \quad x + y^2 - z = 124 \quad \text{is:}$$

**A.** $0$ **B.** $1$ **C.** $2$ **D.** $3$ **E.** none of these.

**13.** Which of the following conditions does *not* guarantee that the convex quadrilateral $ABCD$ is a parallelogram?

**A.** $AB = CD$ and $AD = BC$ **B.** $\angle A = \angle C$ and $\angle B = \angle D$

**C.** $AB = CD$ and $\angle A = \angle C$ **D.** $AB = CD$ and $AB$ is parallel to $CD$ **E.** none of these.

**14.** How many of the expressions

$$x^3 + y^4, \quad x^4 + y^3, \quad x^3 + y^3, \quad \text{and} \quad x^4 - y^4,$$

are positive for all possible numbers $x$ and $y$ for which $x > y$?

**A.** 0 **B.** 1 **C.** 2 **D.** 3 **E.** 4.

**15.** In triangle $ABC$, the altitude from $A$ to $BC$ meets $BC$ at $D$, and the altitude from $B$ to $CA$ meets $AD$ at $H$. If $AD = 4$, $BD = 3$ and $CD = 2$, then the length of $HD$ is

**A.** $\frac{\sqrt{5}}{2}$ **B.** $\frac{3}{2}$ **C.** $\sqrt{5}$ **D.** $\frac{5}{2}$ **E.** $\frac{3\sqrt{5}}{2}$.

**16.** Which of the folowing is the best approximation to

$$\frac{(2^3-1)(3^3-1)(4^3-1)}{(2^3+1)(3^3+1)(4^3+1)} \cdots \frac{(100^3-1)}{(100^3+1)}?$$

**A.** $\frac{3}{5}$ **B.** $\frac{33}{50}$ **C.** $\frac{333}{500}$ **D.** $\frac{3,333}{5,000}$ **E.** $\frac{33,333}{50,000}$.

Last issue we gave the **SHARP** U.K. Intermediate Mathematical Challenge, written February 2, 1995. Here are answers.

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|---|---|---|---|---|---|---|---|---|---|
| **1.** | E | **2.** | C | **3.** | D | **4.** | E | **5.** | A |
| **6.** | B | **7.** | C | **8.** | B | **9.** | D | **10.** | A |
| **11.** | E | **12.** | D | **13.** | C | **14.** | D | **15.** | B |
| **16.** | A | **17.** | A | **18.** | C | **19.** | C | **20.** | D |
| **21.** | D | **22.** | E | **23.** | B | **24.** | B | **25.** | A |

That completes this month's Skoliad Corner. I need materials of a suitable level to build up a bank of contests. Please send me suitable materials as well as comments, criticisms, and suggestions. I would like to have some feedback too about how your students do with these materials.