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Inequities in the Shanks–Rényi Prime Number Race

Let $\pi(x\,;\,q,a)$ denote the number of primes up to x that are congruent to a modulo q. Inequalities of the type $\pi(x\,;\,q,a) > \pi(x\,;\,q,b)$ are more likely to hold if a is a non-square modulo q and b is a square modulo q (the so-called "Chebyshev Bias" in comparative prime number theory). However, the tendencies of the various $\pi(x\,;\,q,a)$ (for nonsquares a) to dominate $\pi(x\,;\,q,b)$ have different strengths. A related phenomenon is that the six possible inequalities of the form $\pi(x\,;\,q,a_1) > \pi(x\,;\,q,a_2) > \pi(x\,;\,q,a_3)$, with a_1, a_2, a_3 all non-squares modulo q, are not equally likely; some orderings are preferred over others. For given values $q, a, b, \ldots$, these tendencies can be quantified and computed, but only using laborious numerical integration of functions involving zeros of the appropriate Dirichlet L-functions. In this talk we describe a framework for explaining which nonsquares a are most dominant for a given square b, for example, based only on elementary properties of the congruence classes a modulo q rather than the complicated computations just mentioned. These elementary properties, on the other hand, do derive from consideration of differences in the distributions of zeros of various L-functions, for example those corresponding to odd and even characters.