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Paul Lévy and the Dichotomy between the Normal and other Stable

The ubiquity of describing the statistics of characteristics in large populations by a normal distribution is commonly accepted. Thus for instance Hernstein and Murray in their widely disseminated book *The Bell Curve* describe the Normal Distribution as:

> "A common way in which natural phenomena arrange themselves approximately."

In fact their model for the distribution of IQ's is mathematically way off the mark in satisfying the criteria for a normal distribution.

A more precise, yet still off the mark definition is in Jim Holt's article in the January 4, 2005, New Yorker Magazine:

> "As a matter of mathematics the Bell Curve is guaranteed to arise whenever some variable is determined by lots of little causes (like human height, health, diet) operating more or less independently."

The great French mathematician Paul Lévy in writing his classic 1924 *Calcul des Probabilités*—in spite of the fact that the eminent mathematicians Borel and Deltheil felt it unnecessary to make the notion of probability more mathematically precise rather than to rely on common sense reasoning—intended to systematically develop and use the method of characteristic functions in order to simplify proofs about limit laws.

A sufficient condition for the sums of a large number of "individually small", independent random variables to approach the normal distribution—*i.e.*, for the Central Limit Theorem to hold—was first given by Liapounoff in 1901 and a more general one by Lindeberg in 1922. Paul Lévy used this simpler method to derive Lindeberg's condition. In so doing he put his finger on the essential necessary condition and its meaning for the Central Limit Theorem to hold. This condition states that the components of the sum be not only "individually negligible" (small) with respect to their total sum, but that they be "uniformly negligible" (we might use the term "collectively negligible"), *i.e.*, the probability that even the largest component random variable be of the order of the magnitude of the sum, must be negligible.

Lévy showed that in case this condition is not satisfied there exist families of limiting distributions for sums of independent random variables, among which the so-called Stable Laws of Index α, where $0 < \alpha < 2$. Here the approach to the limiting distribution is determined by the contribution of the few largest components in the sum. Consequently the probability of values of the sum which deviate from the mean (or the median in case $\alpha < 1$) by a large amount is considerable. The "tail" of the probability distribution of the largest component in the case where the sum converges to a stable distribution of index α, as well as the "tail" of the limiting stable distribution decrease as the function $x^{-\alpha}$. The limiting distribution of the sum reflects that of its largest component terms. This dichotomy defines the "domains of attraction" of the normal vs. those of the other stable distributions.

The statistics of social phenomena in which stable distributions prevail such as wealth, power, batting averages, intellectual accomplishments, physical beauty, *etc.*, are hardly ever discussed in the popular culture where they most emphatically deserve more attention.