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Asterisk topologies on the direct sum of topological Abelian groups

Let $(G_i)_{i\in I}$ be a family of topological Abelian groups and let $\bigoplus_{i\in I} G_i$ denote their algebraic direct sum, that is, the subgroup of the product $\prod_{i\in I} G_i$ formed by those families $x=(x(i))_{i\in I}$ for which $x(i)=0$ for all but finitely many $i\in I$. If we let $H^\wedge$ denote the character group of the Abelian group H, there are canonical algebraic isomorphisms

$$\Big(\prod_{i\in I} G_i\Big)^\wedge \approx \bigoplus_{i\in I} G_i^\wedge \quad \Big(\bigoplus_{i\in I} G_i\Big)^\wedge \approx \prod_{i\in I} G_i^\wedge .$$

Two significant group topologies arise on $\bigoplus_{i\in I} G_i$ in a very natural way: the box topology $\mathcal{T}_b$, given by "rectangular" neighborhoods of zero, and the coproduct topology $\mathcal{T}_f$, which is the finest group topology on $\bigoplus_{i\in I} G_i$ which makes all canonical inclusions continuous.

Nevertheless, in general none of them turns the above-mentioned isomorphisms into topological ones, when considering Tychonoff topology on the products, and compact-open topology on all dual groups. The analysis of such duality properties leads to the definition of an intermediate topology $\mathcal{T}_*$, the so-called *asterisk topology*, firstly introduced by Kaplan in 1948. Actually the lack of a natural generalization of Minkowski functional to groups gives rise to a number of variants of Kaplan's original definition. We shall survey what is known about the conditions under which such topologies are in fact the same, their behaviour with respect to duality, reflexivity and local quasi-convexity, and their relation with each other and with the box and coproduct topologies.