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Some new applications of resolutions

The method of resolutions was introduced in [1] (see also [2]). This method allows us to construct new spaces using given collections of spaces. Many examples of applications of this method are given in [3]. By applying iterated resolutions and fully closed mappings one can obtain more sophisticated examples [4]. Here we present several new applications of resolutions.

Theorem 1 *For any prime p there exists a 2-dimensional homogeneous separable first countable compact space T_p such that $\dim(T_p \times T_q) = 3$ for $p \neq q$.*

Question 1 Are there homogeneous metrizable compacta X and Y such that $\dim(X \times Y) < \dim X + \dim Y$?

Recent results by J. L. Bryant [5] imply that if X and Y are homogeneous metrizable ANR-compacta, then

$$\dim(X \times Y) = \dim X + \dim Y \tag{1}$$

Question 2 Does the equality (1) hold if X is a homogeneous ANR-compactum and Y is an arbitrary (homogeneous) metrizable compactum?

Remark 1 As for Question 1, we cannot omit homogeneity of Y, since Pontryagin's surface Π_2 is homogeneous.

Another two results are joint with A. V. Ivanov and J. van Mill.

Theorem 2 (CH; [6]) *For every $n \in \mathbf{N}$, there exists a family of separable compacta X_i, $i \in \mathbf{N}$, such that for every non-empty finite subset M of $\mathbf{N}$ and every non-empty closed subset F of $\Pi_{i \in M} X_i$ we have $\dim F = k(F)n$, where $k(F)$ is integer such that $k(F) \geq 1$ for infinite F. Moreover, $|F| = 2^c$ for infinite closed F.*

Theorem 3 (CH; [6]) *There exists an infinite separable compactum X such that for any positive integer m, if F is an infinite closed subset of X^m, then $|F| = 2^c$ and F is strongly infinite-dimensional.*

Question 3 Does there exist in ZFC an n-dimensional compactum Y_n, $n \geq 2$, such that for every $m \geq 2$, every non-empty closed subset F of Y_n^m has dimension kn, where k is some integer between 0 and m?

Question 4 Does there exist in ZFC an infinite-dimensional compactum Z such that for every non-empty closed subset F of Z^2 we have either $\dim F = 0$ or F is infinite-dimensional?

References

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