MAXIM BURKE, University of Prince Edward Island, Charlottetown, PE C1A 4P3

Large entire cross-sections of second category sets in $\mathbb{R}^{n+1}$

By the Kuratowski–Ulam theorem, if $A \subseteq \mathbb{R}^{n+1} = \mathbb{R}^n \times \mathbb{R}$ is a Borel set which has second category intersection with every ball (*i.e.*, is "everywhere second category"), then there is a $y \in \mathbb{R}$ such that the section $A \cap (\mathbb{R}^n \times \{y\})$ is everywhere second category in $\mathbb{R}^n \times \{y\}$. If A is not Borel, then there may not exist a large cross-section through A, even if the section does not have to be flat. For example, a variation on a result of T. Bartoszynski and L. Halbeisen shows that there is an everywhere second category set $A \subseteq \mathbb{R}^{n+1}$ such that for any polynomial p in n variables, $A \cap \mathrm{graph}(p)$ is finite. It is a classical result that under the Continuum Hypothesis, there is an everywhere second category set L in $\mathbb{R}^{n+1}$ which has only countably many points in any first category set. In particular, $L \cap \mathrm{graph}(f)$ is countable for any continuous function $f\colon \mathbb{R}^n \to \mathbb{R}$. We prove that it is relatively consistent with ZFC that for any everywhere second category set A in $\mathbb{R}^{n+1}$, there is a function $f\colon \mathbb{R}^n \to \mathbb{R}$ which is the restriction to $\mathbb{R}^n$ of an entire function on $\mathbb{C}^n$ and is such that, relative to $\mathrm{graph}(f)$, the set $A \cap \mathrm{graph}(f)$ is everywhere second category. Moreover, given a non-negative integer k, a function $g\colon \mathbb{R}^n \to \mathbb{R}$ of class C^k and a positive continuous function $\varepsilon\colon \mathbb{R}^n \to \mathbb{R}$, we may choose f so that for all multiindices α of order at most k and for all $x \in \mathbb{R}^n$, $|D^\alpha f(x) - D^\alpha g(x)| < \varepsilon(x)$. The method builds on fundamental work of K. Ciesielski and S. Shelah which provides, for everywhere second category sets in $2^\omega \times 2^\omega$, large sections which are the graphs of homeomorphism of 2^ω.